Notes – PVA

Motion along a Line – Suppose that an object is moving along a coordinate line (say the x-axis) so that we know its position s on that line as a function of time t: $s = f(t)$

The displacement of the object over the time interval from t to $t + \Delta t$ is $\Delta s = f(t + \Delta t) - f(t)$ and the **average velocity** of the object over that time interval is $\dfrac{\text{displacement}}{\text{travel time}} = \dfrac{\Delta s}{\Delta t} = \dfrac{f(t+\Delta t) - f(t)}{\Delta t}$

If we have a position function, the first derivative will give us velocity (instantaneous), and the second derivative will give us acceleration.

DEFINITION	SYMBOLIC RULE	
Instantaneous Velocity – is the derivative of the position function with respect to time.	$v(t) = \dfrac{ds}{dt} = \lim\limits_{\Delta t \to 0} \dfrac{f(t+\Delta t) - f(t)}{\Delta t}$	The earliest questions that motivated the discovery of calculus were concerned with velocity & acceleration, particularly of freely falling bodies under the force of gravity.
Speed – is the absolute value of velocity.	$\text{speed} = \lvert v(t) \rvert = \left\lvert \dfrac{ds}{dt} \right\rvert$	The mathematical description of this type of motion captured the imagination of many great scientists, including Aristotle, Galileo, and Newton. Experimental and theoretical investigations revealed that the distance a body released from rest falls freely is proportional to the square of the amount of time it has fallen. We express this by saying: $s = \frac{1}{2}gt^2$
Acceleration – is the derivative of velocity with respect to time.	$a(t) = \dfrac{dv}{dt} = \dfrac{d^2s}{dt^2}$	**FREE FALL CONSTANTS (on Earth)** English units: $g = 32$ ft/sec^2 $s(t) = \frac{1}{2}(32)t^2 = 16t^2$ Metric units: $g = 9.8$ m/sec^2 $s(t) = \frac{1}{2}(9.8)t^2 = 4.9t^2$

English	Calculus
particle at rest	velocity = 0
particle moving right	velocity > 0
particle moving left	velocity < 0
particle changes direction	velocity changes sign
total distance traveled from time t_1 to t_2(where t_c = time when the particle changes direction)	$\lvert s(t_1)-s(t_c)\rvert + \lvert s(t_c)-s(t_2)\rvert$

Function	*Units*	*Examples / Abbreviations*
$s(t)$	linear units	feet or meters: ft or m
$v(t)$	linear units per unit of time	feet per second: ft/s meters per second: m/s
$a(t)$	linear units per unit of time squared	feet per second squared: ft/s^2 centimeters per second squared: cm/s^2

P V A

- ∞ Particle moves forward when velocity is positive.
- ∞ Particle moves backward when velocity is negative.
- ∞ Particle standing still or at rest when graph of position is horizontal. (think – velocity is zero)
- ∞ Particle moving right when graph of position has a positive slope.
- ∞ Particle moving left when graph of position has a negative slope.
- ∞ Particle moves at its greatest speed when absolute value of velocity is maximized.
- ∞ Particle's acceleration is zero when velocity is constant.
- ∞ Particle's acceleration is negative when velocity is decreasing.
- ∞ Particle's acceleration is positive when velocity is increasing.
- ∞ Particle speeds up when velocity is negative and decreasing or velocity is positive and increasing. vel & accel have same sign
- ∞ Particle slows down when velocity is positive and decreasing or velocity is negative and increasing. vel & accel have different signs

P V A

PVA class notes

1. The position function of a particle moving along the x-axis is given by $x(t) = t^3 - 12t^2 + 36t - 20$ for $0 \le t \le 8$.

   a) Find the velocity and acceleration of the particle.
   b) Find the open interval when the particle is moving to the left. → vel is neg
   c) Find the velocity of the particle when the acceleration is 0.
   d) Describe the motion of the particle.

a) $v(t) = x'(t) = 3t^2 - 24t + 36$

$a(t) = v'(t) = 6t - 24$

b) $3t^2 - 24t + 36 = 0$

$3(t^2 - 8t + 12) = 0$

$3(t-2)(t-6) = 0$

$t = 2 \quad t = 6$

+ | − | +

2 6

$(2, 6)$

c) $6t - 24 = 0$

$t = 4$

$v(4) = 3(4)^2 - 24(4) + 36 = -12$

d)

t	x(t)
0	-20
2	12
6	-20
8	12

At t = 0, the particle is 20 units to the left of the origin. The particle moves 32 to the right for 32 units. At t = 2, the particle moves to the left 32 units over 4 sec. At t = 6 until t = 8, the particle moves 32 units to the right.

2. The position of a particle is given by the equation $s(t) = t^3 - 6t^2 + 9t$ where t is measured in seconds and s is measured in meters.

   a) Find the velocity at time t. $v(t) = 3t^2 - 12t + 9$ m/s
   b) What is the velocity at 2 s? at 4 s? $v(2) = 3(2)^2 - 12(2) + 9 = -3$ m/s $\quad v(4) = 3(4)^2 - 12(4) + 9 = 9$ m/s
   c) When is the particle at rest? vel = 0
   d) When is the particle moving to the right? vel is pos.
   e) Find the total distance traveled by the particle during the first 5 s.

c) $3t^2 - 12t + 9 = 0$

$3(t^2 - 4t + 3) = 0$

$3(t-3)(t-1) = 0$

$t = 3$ sec $\quad t = 1$ sec

d) + | − | +

1 3

$(0, 1)$ and $(3, \infty)$

e)

t	s(t)
0	0
1	4
3	0
5	20

28 m

3. Suppose that a ball is dropped from the upper observation deck of the CN Tower in Toronto, 450 m above the ground. $s(t) = -\frac{1}{2}gt^2 + v_0 t + h_0$

   a) Write a height equation. $s(t) = -4.9t^2 + 450$
   b) Find the average velocity of the ball from t = 3 seconds to t = 5 seconds.
   c) Find the velocity of the ball at 5 seconds.
   d) When will the ball hit the ground? ht = 0
   e) With what velocity will the ball hit the ground?

b) avg. vel. $= \frac{\Delta \text{ pos}}{\Delta \text{ time}}$

$(3, 405.9) \quad (5, 327.5)$

$\frac{327.5 - 405.9}{5 - 3} = -39.$

c) $v(t) = -9.8t$

$v(5) = -9.8(5) = -49$ m/s

e) $v(9.583) = -9.8(9.583)$

-93.915 m/s

d) $-4.9t^2 + 450 = 0$

$t^2 = \frac{4500}{49} \quad t = 9.583$ sec