AP Calculus BC Review   Notes Riemann Sums and Trapezoidal Sum

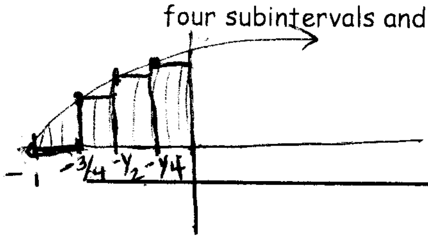
You should be able to . . .

- **Approximate the area under a curve using a Riemann or Trapezoidal Sum**
- **Recognize the definition of a Riemann Sum**

A *Riemann Sum*, denoted $\sum_{i=1}^{n} f(x_i)\Delta x$, is the sum of n contiguous rectangles along the interval $[a, b]$ with bases of uniform width, Δx, equal to $\frac{b-a}{n}$ and with heights, $f(x_i)$, where x_i is some arbitrary point within each closed subinterval represented by the bases.

$$\boxed{\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x, \text{where } \Delta x = \frac{b-a}{n}}$$

Ex 1) Approximate the area under the curve $f(x) = \sqrt{x+1}$, $-1 \le x \le 0$ with a Riemann Sum using four subintervals and left endpoints.



width $= \frac{0-(-1)}{4} = \frac{1}{4}$

$\frac{1}{4}\left(\sqrt{0} + \sqrt{\frac{1}{4}} + \sqrt{\frac{1}{2}} + \sqrt{\frac{3}{4}}\right)$

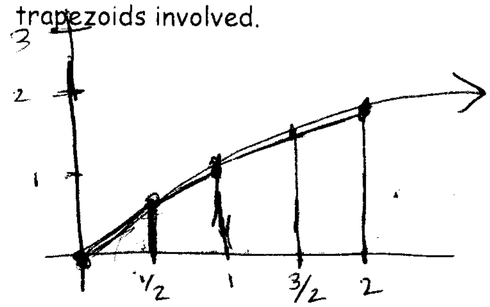
$= .518$

Trapezoidal Rule

$$T = \frac{h}{2}(y_0 + 2y_1 + 2y_2 + \cdots + 2y_{n-1} + y_n) \quad \textit{where } h = \text{width of subinterval}$$

This gives us a better approximation than LRAM or RRAM

Ex 2) Approximate $\int_0^2 \sqrt{x}dx$ with 4 trapezoids. Sketch a figure showing the curve and the trapezoids involved.



ht $= \frac{b-a}{h} = \frac{2-0}{4} = \frac{1}{2}$

$\frac{1}{2} \cdot \frac{1}{2}\left(\sqrt{0} + 2\sqrt{1/2} + 2\sqrt{1} + 2\sqrt{\frac{3}{2}} + \sqrt{2}\right)$

1.819

Ex 3) Multiple-choice Which of the following integral expressions is equal to $\lim_{n\to\infty} \sum_{k=1}^{n} \left(\sqrt{1+\frac{3k}{n}} \frac{1}{n}\right)$? b-a

A) $\int_0^1 \sqrt{1+3x}dx$ B) $\int_0^3 \sqrt{1+x}dx$ C) $\int_1^4 \sqrt{x}dx$ D) $\frac{1}{3}\int_0^3 \sqrt{x}dx$

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Ex 4) Free Response

A hot cup of coffee is taken into a classroom and set on a desk to cool. The table shows the rate $R(t)$ at which temperature of the coffee is dropping at various times over an eight minute period, where $R(t)$ is measured in degrees Fahrenheit per minute and t is measured in minutes. When $t = 0$, the temperature of the coffee is 113° F.

t (minutes)	0	3	5	8
$R(t)$ (° F/min.)	5.5	2.7	1.6	0.8

(a) Estimate the temperature of the coffee at $t = 8$ minutes by using a left Riemann sum with three subintervals and values from the table. Show the computations that lead to your answer.

(b) Use values from the table to estimate the average rate of change of $R(t)$ over the eight minute period. Show the computations that lead to your answer.

(c) A model for the rate at which the temperature of the coffee is dropping is given by the function $y(t) = 7e^{-0.3t}$, where $y(t)$ is measured in degrees Fahrenheit per minute and t is measured in minutes. Use the model to find the temperature of the coffee at $t = 8$ minutes.

(d) Use the model given in (b) to find the average rate at which the temperature of the coffee is dropping over the eight minute period.

a) $3(5.5) + 2(2.7) + 3(1.6) = 26.7°$

$113° - 26.7° = 86.3°$

b) $(0, 5.5)\ (8, 0.8)$

$\frac{0.8-5.5}{8-0} = -.5875\ °F/min/min$

$= -.588\ °F/min^2$

c) $\int_0^8 -7e^{-0.3t}\,dt = T(8) - T(0)$

$T(0) +$

$113° + {}^{-}21.217° = 91.783°F$

d) $\frac{1}{8-0}\int_0^8 -7e^{-0.3t}\,dt = -2.652$

When $t = 8$ min, the temp is dropping at a rate of $2.652\ °F/min$

Ex 5) Free Response

A metal wire of length 8 centimeters is heated at one end. The table below gives selected values of the temperature $T(x)$, in degrees Celsius, of the wire x cm from the heated end.

Distance x (cm)	0	1	5	6	8
Temperature $T(x)$ (°C)	100	93	70	62	55

(a) Estimate $T'(7)$. Show the work that leads to your answer. Indicate units of measure.

(b) Write an integral expression in terms of $T(x)$ for the average temperature of the wire. Estimate the average temperature of the wire using a trapezoidal sum with the four subintervals indicated by the data in the table. Indicate units of measure.

(c) Find $\int_0^8 T'(x)\,dx$, and indicate units of measure. Explain the meaning of $\int_0^8 T'(x)\,dx$ in terms of the temperature of the wire.

a) $(6, 62)(8, 55)$

$\frac{55-62}{8-6} = -3.5\ °C/cm$

b) $\frac{1}{8-0}\int_0^8 T(x)\,dx$

$\frac{1}{8}\left[\frac{1}{2}(1)(100+93) + \frac{1}{2}(4)(93+70) + \frac{1}{2}(1)(70+62) + \frac{1}{2}(2)(62+55)\right]$

$\frac{1}{8}(605.5) = 75.6875\ °C$

c) $\int_0^8 T'(x)\,dx = T(8) - T(0)$

$= 55 - 100$

$= -45°C$

from one end to the other, the temp. has dropped 45°C.