You should be able to . . .

- **Find the derivative or second derivative of polar or parametric equations**

Parametric Equations

1st Derivative: $\dfrac{dy}{dx}=\dfrac{\frac{dy}{dt}}{\frac{dx}{dt}}$, provided $\dfrac{dx}{dt}\neq 0$

To find the second derivative of a parametrized curve, we find the derivative of the first derivative:

$$\frac{d^2y}{dx^2}=\frac{d}{dx}(y')=\frac{\frac{dy'}{dt}}{\frac{dx}{dt}}$$

1. Find the first derivative (dy/dx).
2. Find the derivative of dy/dx with respect to t.
3. Divide by dx/dt.

Polar Equation

By writing the Polar equations in parametric form, we can derive the following:

$$\frac{dy}{dx}=\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}=\frac{\frac{d}{d\theta}r\sin\theta}{\frac{d}{d\theta}r\cos\theta}=\boxed{\frac{r'\sin\theta+r\cos\theta}{r'\cos\theta-r\sin\theta}}$$

We use the product rule here.

- **Find horizontal or vertical tangents to a polar or parametric graph**
- **Interpret derivatives in the context of a situation for a polar or parametric graph**

Speed = $\sqrt{\left(\dfrac{dx}{dt}\right)^2+\left(\dfrac{dy}{dt}\right)^2}$

Ex 1) Given $\begin{matrix}x=t^3+1\\ y=t^2+t\end{matrix}$. Find the equation of the tangent line when t = 3.

point $(x,y)=(28,12)$

Slope $=\dfrac{dy}{dx}=\dfrac{\frac{dy}{dt}}{\frac{dx}{dt}}=\dfrac{2t+1}{3t^2}\Big|_{t=3}=\dfrac{7}{27}$

$y-12=\frac{7}{27}(x-28)$

Ex 2) An object is at $\langle \cos t, \sin t \rangle$ for any time t. Show that the object is traveling at a constant speed.

$$\frac{dx}{dt} = -\sin t \qquad \frac{dy}{dt} = \cos t$$

$$\text{Speed} = \sqrt{(-\sin t)^2 + (\cos t)^2} = \sqrt{\sin^2 t + \cos^2 t} = \sqrt{1} = 1$$

Ex 3) Find the $\underbrace{\text{slope of a tangent line}}_{\text{deriv.}}$ to the curve $r = 3 - 4\sin\theta$.

$$x = r\cos\theta = (3-4\sin\theta)\cos\theta$$
$$y = r\sin\theta = (3-4\sin\theta)\sin\theta$$
$$= 3\sin\theta - 4\sin^2\theta$$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{(3-4\sin\theta)(\cos\theta) + (\sin\theta)(-4\cos\theta)}{(3-4\sin\theta)(-\sin\theta) + (\cos\theta)(-4\cos\theta)}$$

deriv. = 0

Ex 4) Find the points where the graph of $r = 5 - 5\sin\theta$ has horizontal tangents.

$$x = (5-5\sin\theta)\cos\theta$$
$$y = (5-5\sin\theta)\sin\theta$$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$$

$$= \frac{(5-5\sin\theta)\cos\theta + \sin\theta(-5\cos\theta)}{(5-5\sin\theta)(-\sin\theta) + \cos\theta(-5\cos\theta)}$$

$$5\cos\theta - 5\sin\theta\cos\theta - 5\sin\cos\theta = 0$$

$$-10\sin\theta\cos\theta + 5\cos\theta = 0$$

$$-5\cos\theta(2\sin\theta - 1) = 0$$

$$-5\cos\theta = 0 \qquad 2\sin\theta - 1 = 0$$

$$\theta = \cos^{-1}(0) \qquad \theta = \sin^{-1}\left(\tfrac{1}{2}\right)$$

$$\cancel{\tfrac{\pi}{2}}, \ \tfrac{3\pi}{2} \qquad \tfrac{\pi}{6}, \ \tfrac{5\pi}{6}$$

$(0, -10)$ $\left(-\frac{5\sqrt{3}}{4}, \frac{5}{4}\right)$

$\left(2.5\frac{\sqrt{3}}{2}, \frac{5}{4}\right)$

$= \left(\frac{5\sqrt{3}}{4}, \frac{5}{4}\right)$