

Partial Fractions (non-repeated linear factors)

* degree of numerator < degree of denominator
if not, use long division first

$$\text{EX1} \quad \int \frac{7x+6}{x^2+5x+6} dx = \int \left(\frac{15}{x+3} + \frac{-8}{x+2} \right) dx$$

$$\frac{7x+6}{x^2+5x+6} = \frac{A}{x+3} + \frac{B}{x+2}$$

$(x+3)(x+2)$

$$15 \ln|x+3| - 8 \ln|x+2| + C$$

$$7x+6 = A(x+2) + B(x+3)$$

$$\text{Let } x = -2 : -8 = A(0) + B(1)$$

$$B = -8$$

$$\text{Let } x = -3 : -15 = A(-1) + B(0)$$

$$A = 15$$

$$\text{EX2} \quad \int \frac{5x+3}{x^3-2x^2-3x} dx = \int \frac{5x+3}{x(x-3)(x+1)} dx$$

$x(x^2-2x-3)$

$$\frac{5x+3}{x(x-3)(x+1)} = \frac{A}{x} + \frac{B}{x-3} + \frac{C}{x+1}$$

$$5x+3 = A(x-3)(x+1) + B(x)(x+1) + C(x)(x-3)$$

$$\underline{x=0}: 3 = A(-3)(1) + B(0)(1) + C(0)(-3)$$

$$3 = -3A$$

$$A = -1$$

$$x=3: 10 = A(0)(4) + B(3)(4) + C(3)(0)$$

$$10 = 12B$$

$$B = \frac{10}{12} = \frac{5}{6}$$

$$x=-1: -2 = A(-4)(0) + B(-1)(0) + C(-1)(-4)$$

$$-2 = 4C$$

$$C = -\frac{2}{4} = -\frac{1}{2}$$

$$\int \left(\frac{-1}{x} + \frac{\frac{5}{6}}{x-3} + \frac{-\frac{1}{2}}{x+1} \right) dx$$

$$\boxed{-1 \ln|x| + \frac{5}{6} \ln|x-3| - \frac{1}{2} \ln|x+1| + C}$$

$$\underline{\text{EX 3}} \int \frac{x^4 + 8x^2 + 8}{x^3 - 4x} dx = \int \left(x + \frac{12x^2 + 8}{x^3 - 4x} \right) dx$$

$$\begin{array}{r} x^3 - 4x \overline{) x^4 + 0x^3 + 8x^2 + 0x + 8} \\ \underline{-(x^4 \quad -4x^2)} \\ 12x^2 + 8 \end{array}$$

$\frac{1}{2}x^2$
partial fractions

$$\frac{12x^2 + 8}{x(x-2)(x+2)} = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{x+2}$$

$$12x^2 + 8 = A(x-2)(x+2) + B(x)(x+2) + C(x)(x-2)$$

$$x=0: 8 = -4A$$

$$A = -2$$

$$x=2: 56 = 8B$$

$$B = 7$$

$$x=-2: 56 = 8C$$

$$C = 7$$

$$\frac{1}{2}x^2 + \int \left(\frac{-2}{x} + \frac{7}{x-2} + \frac{7}{x+2} \right) dx$$

$$\frac{1}{2}x^2 - 2\ln|x| + 7\ln|x-2| + 7\ln|x+2| + C$$

EX 4

$$\int \frac{x-13}{(2x-1)(x-3)} dx$$

$$= \int \left(\frac{5}{2x-1} + \frac{-2}{x-3} \right) dx$$

$$\frac{x-13}{(2x-1)(x-3)} = \frac{A}{2x-1} + \frac{B}{x-3}$$

$$x-13 = A(x-3) + B(2x-1)$$

$$x=3: -10 = 5B$$

$$B = -2$$

$$x = \frac{1}{2}: -\frac{25}{2} = -\frac{5}{2}A$$

$$A = \frac{-25}{2} \cdot \frac{2}{5} = 5$$

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$$u = 2x-1$$

$$\frac{du}{dx} = 2$$

$$\frac{1}{2} du = dx$$

$$5 \cdot \frac{1}{2} \int \frac{du}{u}$$

$$\frac{5}{2} \ln|2x-1| - 2 \ln|x-3| + C$$