

Factorial

$$n! = n(n-1)(n-2)(n-3) \dots (1)$$

Example 1 Find:

A. $6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720$

B. $8! = 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 40,320$

C. $0! = 1$

D. $\frac{8!}{5!} = \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 336$

E. $\frac{31(4!)}{7!} = \frac{3 \cdot 2 \cdot 1 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 35$

A Permutation is an ordered arrangement of objects of a set. The objects cannot be repeated.

$${}^n P_r = \frac{n!}{(n-r)!}$$

total \leftarrow items taken from n items \leftarrow pick

Example 2 In how many ways can 5 airplanes line up on a runway? $5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

$${}^5 P_5 = \frac{5!}{(5-5)!} = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{1} = 120$$

Example 3 You have 20 CD's. You are asked to list your five favorite ones in order of preference. In how many ways can the five favorites be ranked?

$${}^{20} P_5 = \frac{20!}{(20-5)!} = \frac{20 \cdot 19 \cdot 18 \cdot 17 \cdot 16 \cdot 15!}{15!} = 1,860,480$$

A Combination is an arrangement of objects in which order doesn't matter and items can be repeated

$${}^n C_r = \frac{n!}{(n-r)!r!}$$

r items taken from n items

Example 4 Given 5 people, choose a group of 2 for a committee. In how many ways can a committee be formed?

$${}^5C_2 = \frac{5!}{2!3!} = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1 \cdot 3 \cdot 2 \cdot 1} = 10$$

Example 5 Given 20 male actors, select 4 for the roles of Mark, Roger, Angel, and Collins.

$${}^{20}P_4 = 116,280$$

Example 6 Given 20 actors, select 4 to attend a workshop.

$${}^{20}C_4 = 4,845$$

Example 7 How many different 4-card hands can be dealt from a 52-card deck?

$${}^{52}C_4 = 270,725$$

Example 8 Florida's lottery game, LOTTO, is set up so that each player chooses 6 different numbers from 1 to 53. If the 6 numbers chosen match the 6 numbers drawn randomly, the player wins the top cash prize. With one LOTTO ticket, what is the probability of winning this prize?

hint: $p(\text{winning}) = \frac{\text{number of ways of winning}}{\text{total number of possible winning combinations}}$

$$= \frac{{}^{53}C_6}{1} = \frac{22,957,480}{1} = 22,957,480$$

$$4 \cdot 356 \times 10^8$$

$$0.00000014356$$

Example 9 A club consists of five men and seven women. Three members are selected at random to attend a conference. Find the probability that the selected group consists of:

A. three men

B. one man and two women

$$B. p(\text{1 man \& 2 women}) = \frac{{}^5C_1 \cdot {}^7C_2}{{}^{12}C_3} = \frac{5 \cdot 21}{220} = \frac{105}{220} = \frac{21}{44} = 0.477$$

$$A. p(\text{3 men}) = \frac{\text{\# ways to get 3 men}}{\text{total \# combos of people}}$$

$$\frac{{}^5C_3}{1} = \frac{10}{1} = 10$$

$$\frac{{}^{12}C_3}{{}^5C_3} = \frac{220}{10} = 22$$

$$\frac{22}{23} = 0.956$$