tangent lines at the pole

If $f(\theta) = 0$ (and $f'(\theta) \neq 0$) then the line $\theta = \alpha$ is tangent at the pole to the graph of $r = f(\theta)$

$r = 2\cos 3\theta$ rose 3 petals

$0 \leq \theta < \pi$

$0 \leq 3\theta < 3\pi$

$$2\cos 3\theta = 0$$

$$\cos 3\theta = 0$$

$$3\theta = \cos^{-1}(0)$$

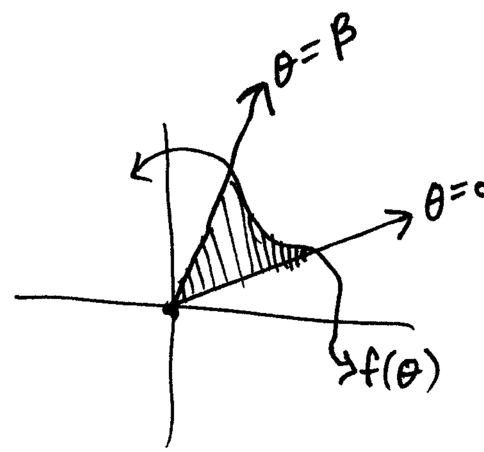
$$3\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$$

$$\theta = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$$

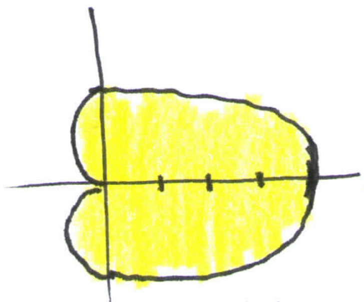
Polar Area

Given $r = f(\theta)$ over $[\alpha, \beta]$ with the radial lines $\theta = \alpha$ and $\theta = \beta$. The area of the region is

$$A = \frac{1}{2}\int_{\alpha}^{\beta} [f(\theta)]^2 \, d\theta$$



EX1 $r = 2+2\cos\theta$. Find the enclosed area.



$2+2\cos\theta = 0$

$\cos\theta = -1$

$\theta = \cos^{-1}(-1)$

$\theta = \pi$

$$A = \frac{1}{2}\int_0^{2\pi} (2+2\cos\theta)^2 \, d\theta$$

18.849 or 18.850

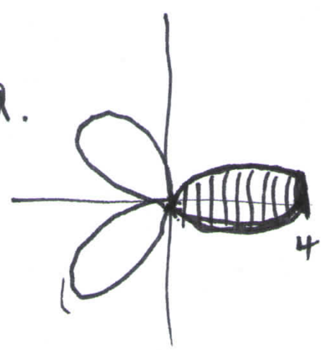
$$2 \cdot \frac{1}{2}\int_0^{\pi} (2+2\cos\theta)^2 \, d\theta$$

EX2 $r = 4\cos 3\theta$ rose 3 petals petal length = 4

a. Find the area enclosed by 1 petal.

b. Find the area enclosed by the graph.

a.



$4\cos 3\theta = 0$ $\qquad 0 \le \theta \le \pi$

$\cos 3\theta = 0$ $\qquad 0 \le 3\theta \le 3\pi$

$3\theta = \cos^{-1}(0)$

$3\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$

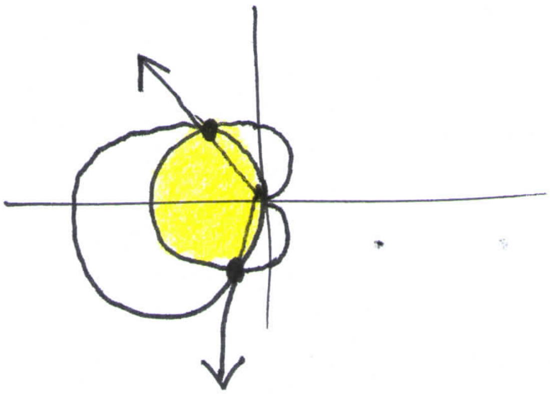
$\theta = \frac{\pi}{6}, \underset{\downarrow \, \pi/2}{\frac{3\pi}{6}}, \frac{5\pi}{6}$

$$A = 2 \cdot \frac{1}{2}\int_0^{\pi/6} (4\cos 3\theta)^2 \, d\theta = 4.188 \text{ or } 4.189$$

$$\frac{1}{2}\int_{-\pi/6}^{\pi/6} (4\cos 3\theta)^2 \, d\theta$$

b. 3* part A = 12.566

EX3 Find the area of the region common to $r = -6\cos\theta$ and $r = 2 - 2\cos\theta$.



$$2 - 2\cos\theta = -6\cos\theta$$
$$2 = -4\cos\theta$$
$$-\frac{1}{2} = \cos\theta$$
$$\theta = \cos^{-1}\left(-\tfrac{1}{2}\right)$$
$$\theta = \frac{2\pi}{3},\ \frac{4\pi}{3}$$

$$-6\cos\theta = 0$$
$$\cos\theta = 0$$
$$\theta = \cos^{-1}(0)$$
$$\theta = \frac{\pi}{2},\ \frac{3\pi}{2}$$

Small region $\frac{1}{2}\int_{\pi/2}^{2\pi/3} (-6\cos\theta)^2\, d\theta = .8152746634$

$* 2$

1.630549327

large region $\frac{1}{2}\int_{2\pi/3}^{4\pi/3} (2-2\cos\theta)^2\, d\theta = 14.07741394$

15.708

Polar Arc Length $\int_{\alpha}^{\beta} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$

EX4 $r = 2 - 2\cos\theta$, $[0, 2\pi]$

$$\int_0^{2\pi} \sqrt{(2-2\cos\theta)^2 + (2\sin\theta)^2}\, d\theta = 16$$