#1 a) $\frac{1}{2}\int_0^{\pi/3} (1-2\cos\theta)^2 \, d\theta$

$1-2\cos\theta = 0$
$\cos\theta = \frac{1}{2}$
$\theta = \cos^{-1}\left(\frac{1}{2}\right)$
$\pi/3$

b) $x = r\cos\theta = (1-2\cos\theta)\cos\theta$
$y = r\sin\theta = (1-2\cos\theta)\sin\theta$

$$\frac{dx}{d\theta} = (1-2\cos\theta)(-\sin\theta) + (\cos\theta)(2\sin\theta)$$

$$\frac{dy}{d\theta} = (1-2\cos\theta)(\cos\theta) + (\sin\theta)(2\sin\theta)$$

c) $$\text{slope} = \frac{dy}{dx} = \left.\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}\right|_{\theta=\pi/2} = \frac{(1-2\cdot 0)(0) + (1)(2\cdot 1)}{(1-2\cdot 0)(-1) + (0)(2\cdot 1)}$$

$$= \frac{2}{-1} = -2$$

point $(0, 1)$

$x = (1-2\cdot 0)\,0 = 0$
$y = (1-2\cdot 0)\,1 = 1$

$y - 1 = -2(x - 0)$
$y = -2x + 1$

#2 a) $\frac{240^\circ}{360^\circ}$ of circle

$$\frac{2}{3}\left(\pi(2)^2\right) + \frac{1}{2}\int_{2\pi/3}^{4\pi/3}\left(3+2\cos\theta\right)^2 d\theta = 10.370$$

b) $r = 3+2\cos\theta$

$$\left.\frac{dr}{dt}\right|_{\theta=\pi/3} = \left.\frac{dr}{d\theta}\right|_{\theta=\pi/3} = \left.-2\sin\theta\right|_{\theta=\pi/3} = -2\left(\frac{\sqrt{3}}{2}\right)$$

$$= -\sqrt{3}$$

$$-1.732$$

when $\theta = \frac{\pi}{3}$, the particle is moving closer to the pole. Since $\frac{dr}{dt}$ is negative.

c) $$\frac{dy}{dt} = \left.\frac{dy}{d\theta}\right|_{\theta=\pi/3} = \left.(3+2\cos\theta)(\cos\theta) + (\sin\theta)(-2\sin\theta)\right|_{\theta=\pi/3}$$

$$= \left(3+2\cdot\frac{1}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(-2\cdot\frac{\sqrt{3}}{2}\right)$$

$y = r\sin\theta = (3+2\cos\theta)(\sin\theta)$

$$= 2 + -\frac{3}{2} = \frac{1}{2}$$

when $\theta = \frac{\pi}{3}$, the particle is moving away from the x-axis since $\frac{dy}{dt} > 0$.