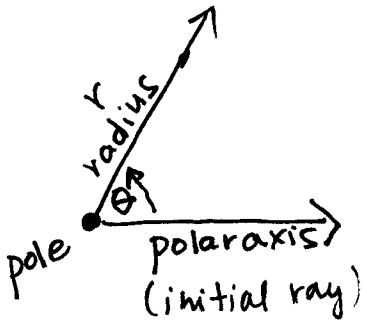
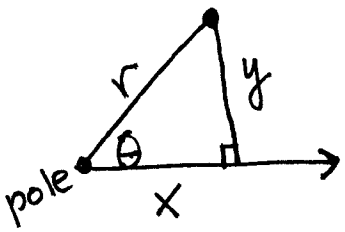
Polar review



(r, θ)



$x^2 + y^2 = r^2$

$r = \sqrt{x^2 + y^2}$

$\tan\theta = \frac{y}{x} \quad \theta = \tan^{-1}\left(\frac{y}{x}\right)$

(x, y) rect → (r, θ) polar

$\cos\theta = \frac{x}{r}$

$\sin\theta = \frac{y}{r}$

$x = r\cos\theta$

$y = r\sin\theta$

(r, θ) polar → (x, y) rect

polar slope

polar eqn $r = f(\theta)$

$x = r\cos\theta = f(\theta)\cdot\cos\theta$

$y = r\sin\theta = f(\theta)\cdot\sin\theta$

$$\text{slope} = \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{f(\theta)\cdot\cos\theta + \sin\theta \cdot f'(\theta)}{-f(\theta)\cdot\sin\theta + \cos\theta \cdot f'(\theta)}$$

$$\frac{dx}{d\theta} \neq 0$$

EX1 $r = 2(1-\sin\theta)$

$y = 2(1-\sin\theta)\cdot\sin\theta$

$x = 2(1-\sin\theta)\cdot\cos\theta$

a) Find $\frac{dy}{dx}$

b) Find the slope at $\left(4, \frac{3\pi}{2}\right)$.

a) $$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{2(1-\sin\theta)\cdot(+\cos\theta) + \sin\theta\cdot 2(-\cos\theta)}{2(1-\sin\theta)\cdot(-\sin\theta) + \cos\theta\cdot 2(-\cos\theta)}$$

b) $$\left.\frac{dy}{dx}\right|_{\theta=\frac{3\pi}{2}} = \frac{2(1-(-1))\cdot(0) + (-1)\cdot 2(-0)}{2(1-(-1))\cdot(-1) + 0\cdot 2(-0)}$$

$$= \frac{0}{4} = 0$$ horiz. tang.