ICM Notes on Polynomials

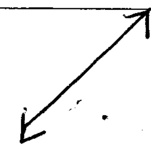
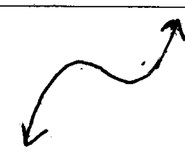
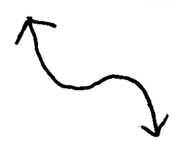
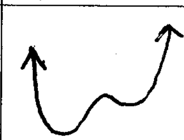
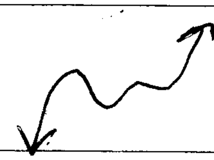
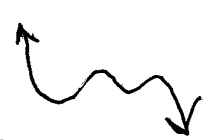
A **polynomial** has form $a_nx^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots + a_1x + a_0$

- the powers n, n-1, etc. are positive integers
- a_nx^n is the leading term
- a_n is the leading coefficient and $\neq 0$
- a_n, a_{n-1}, etc. are real numbers

examples of polynomials: $x^3 - 2x + 7$ $\quad \frac{1}{3}x^4 - 5$

these are not polynomials: $\frac{1}{x^2}$ $\quad \sqrt{x} + 2$

general forms of polynomials

name	**general equation**	**degree**	**graph** pos leading coefficient	neg leading coefficient
constant	$y = \#$	0		
linear	$y = x$	1		
quadratic	$y = x^2$	2		
cubic	$y = x^3$	3		
quartic	$y = x^4$	4		
quintic	$y = x^5$	5		

To determine the end behavior look at limit of f(x) as $x \to -\infty$ and limit of f(x) as $x \to \infty$. In other words, determine where the ends of the graph are headed.

Example 1 Describe the end behavior.

A. $f(x) = 2x^3 + x^2 - 7$ pos

cubic 

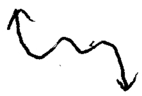
$\lim_{x\to -\infty} f(x) = -\infty$ $\lim_{x\to \infty} f(x) = \infty$

B. $g(x) = -x^4 + 2x^3 + 3x - 3$ neg

quartic 

$\lim_{x\to -\infty} g(x) = -\infty$ $\lim_{x\to \infty} g(x) = -\infty$

C. $h(x) = 24x - 4x^5 + 2x^2 - 7$ neg

quintic 

$\lim_{x\to -\infty} h(x) = \infty$ $\lim_{x\to \infty} h(x) = -\infty$

Finding zeros

repeated roots = "multiplicity"

- **if odd—the graph crosses the x-axis once**
- **if even—the graph touches the x-axis but doesn't cross through (it's tangent)**

powers on factors

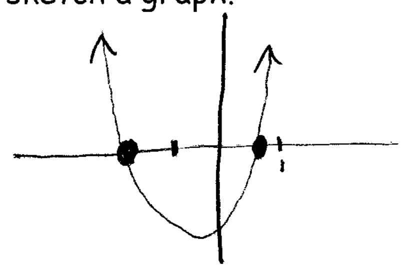
Example 2 Find the roots algebraically and sketch a graph.

A. $f(x) = 3x^2 + 4x - 4$

$(3x-2)(x+2) = 0$

$3x-2=0$ $x+2=0$

$x = 2/3$ $x = -2$



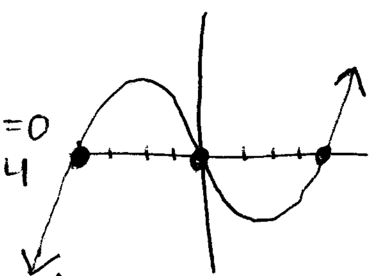
B. $g(x) = x^3 - 16x$

$x(x^2-16)$

$x(x-4)(x+4) = 0$

$x=0$ $x-4=0$ $x+4=0$

$x=4$ $x=-4$



C. $f(x) = x^3 + 4x$

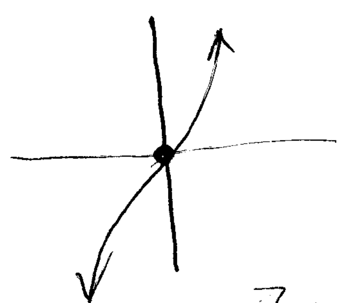
$x(x^2+4) = 0$

$x=0$ $x^2+4=0$

$x^2 = -4$

$x = \pm\sqrt{-4}$

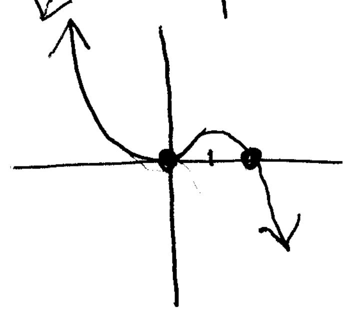
$x = \pm 2i$



D. $y = -x^2(x-2)^1$

$x=0$ $x=2$

multiplicity of 2

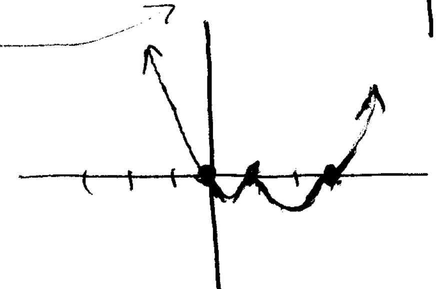


E. $g(x) = (x-3)^3(x-1)^2 x^1$ deg. 6 pos. coeff.

$x-3=0$ $x-1=0$ $x=0$

$x=3$ $x=1$

mult. of 3 mult. of 2



Example 3 Use a calculator to find the roots: $f(x) = -x^3 + 3x^2 + 7x - 2$

-1.726

0.259

4.467

Remainder Theorem
If $f(x)$ is divided by $x - k$, then the remainder is $f(k)$.

Factor Theorem
If $f(k) = 0$, then $x - k$ is a factor of $f(x)$.

Example 4 $p(x) = 2x^2 + 3x + 4$ Divide by $x - 3$.

Example 5 $p(x) = 2x^3 - 3x^2 - 5x - 12$ Is $x - 3$ a factor?

Example 6 $p(x) = 3x^3 + 4x^2 - 5x - 2$ Is $x + 2$ a factor? Is $x + 1$ a factor?

Example 7 $p(x) = x^3 - 3x - 2$ Is $x - 2$ a factor?