Notes – PVA

<u>Motion along a Line</u> – Suppose that an object is moving along a coordinate line (say the x-axis) so that we know its position s on that line as a function of time t: s = f(t)

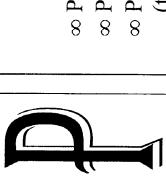
The <u>displacement</u> of the object over the time interval from t to $t + \Delta t$ is $\Delta s = f(t + \Delta t) - f(t)$ and the average velocity of the object over that time interval is $\frac{\text{displacement}}{\text{travel time}} = \frac{\Delta s}{\Delta t} = \frac{f(t + \Delta t) - f(t)}{\Delta t}$

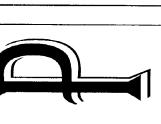
If we have a position function, the first derivative will give us velocity, and the second derivative will give us acceleration.

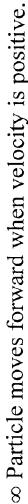
DEFINITION	SYMBOLIC RULE	
Instantaneous Velocity – is the derivative of the position function with respect to time.	$v(t) = \frac{ds}{dt} = \lim_{\Delta t \to 0} \frac{f(t + \Delta t) - f(t)}{\Delta t}$	The earliest questions that motivated the discovery of calculus were concerned with velocity & acceleration, particularly of freely falling bodies under the force of gravity. The mathematical description of this type of motion
Speed – is the absolute value of velocity.	$speed = v(t) = \frac{ds}{dt}$	captured the imagination of many great scientists, including Aristotle, Galileo, and Newton. Experimental and theoretical investigations revealed that the distance a body released from rest falls freely is proportional to the square of the amount of time it has fallen. We express this by saying: $s = \frac{1}{2}gt^2$
Acceleration – is the derivative of velocity with respect to time.	$a(t) = \frac{dv}{dt} = \frac{d^2s}{dt^2}$	FREE FALL CONSTANTS (on Earth) English units: $g = 32$ ft/sec ² $s(t) = \frac{1}{2}(32)t^2 = 16t^2$ Metric units: $g = 9.8$ m/sec ² $s(t) = \frac{1}{2}(9.8)t^2 = 4.9t^2$

English	<u>Calculus</u>
particle at rest	velocity = 0
particle moving right	velocity > 0
particle moving left	velocity < 0
particle changes direction	velocity changes sign
total distance traveled from time t ₁ to t ₂ (where t _c = time when the particle changes direction)	$ s(t_1)-s(t_c) + s(t_c)-s(t_2) $

Function	Units	Examples / Abbreviations
s(t)	linear units	feet or meters: ft or m
V(i)	linear units per unit of time	feet per second: ft/s meters per second: m/s
a(t)	linear units per unit of time squared	feet per second squared: ft/s² centimeters per second squared: cm/s²







 ∞ Particle moves backward when velocity is negative.

 ∞ Particle standing still or at rest when graph of position is horizontal. (think – velocity is zero)

 ∞ Particle moving right when graph of position has a positive slope.

 ∞ Particle moving left when graph of position has a negative slope.

∞ Particle moves at its greatest speed when absolute value of velocity is maximized.

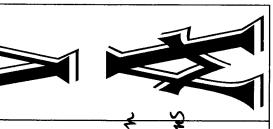
 ∞ Particle's acceleration is zero when velocity is constant.

 ∞ Particle's acceleration is negative when velocity is decreasing.

 ∞ Particle's acceleration is positive when velocity is increasing.

velocity is positive and increasing. accel. ?, vel. have the same sign ∞ Particle speeds up when velocity is negative and decreasing or

velocity is negative and increasing. accel & Vel have different signs ∞ Particle slows down when velocity is positive and decreasing or



√A class notes

- 1. The position function of a particle moving along the x-axis is given by $x(t) = t^3 12t^2 + 36t 20$ for $0 \le t \le 8$.
 - Find the velocity and acceleration of the particle.
 - Find the open interval when the particle is moving to the left.
 - Find the velocity of the particle when the acceleration is 0.
 - d) Describe the motion of the particle.

a)
$$v(t) = x'(t) = 3t^2 - 24t + 36$$

 $a(t) = v'(t) = x''(t) = 6t - 24$

b)
$$3t^2 - 24t + 36 = 0$$
 $\frac{+}{2} + \frac{+}{4}$
 $3(t^2 - 8t + 12) = 0$ moving to les

 $V(4) = 3(4)^2 - 24(4) + 36 = -12$

c) 6t-24=0

+= 4

The particle is 20 units to the left 2 12 of the origin at 6-20 t=0. The particle moves to the right 8 12 32 units until t=2 of the origin at 32 units until t=2. Between t= 2 and 6

3(t-6)(t-2)=0 moving to left => v(t) is negative and by the particle moves the particle moves to the particle is given by the equation $s(t)=t^3-6t^2+9t$ where t is measured in seconds and s is Between t=6 and 8. measured in meters.

- a) Find the velocity at time t. $v(t) = 5'(t) = 3t^2 12t + 9$ m/s
- b) What is the velocity at 2 s? at 4 s? $V(3) = 3(2)^2 12(2) + 9 = -3 \text{ m/s}$ c) When is the particle at rest? $V(4) = 3(4)^2 12(4) + 9 = 9 \text{ m/s}$
- d) When is the particle moving to the right?
- e) Find the total distance traveled by the particle during the first 5 s.

c)
$$3t^{2}-12t+9=0$$

 $3(t^{2}-4t+3)=0$
 $3(t-3)(t-1)=0$
 $t=3$
 $t=1$ sec

d) moving right
$$\Rightarrow$$
 vel is pos.
 $\frac{+}{1}$ $\frac{-}{3}$
(0,1) and (3,00)

28m

the particle moves

to the right 32 units.

- 3. Suppose that a ball is dropped from the upper observation deck of the CN Tower in Toronto, 450 m above the ground. $S(t) = -\frac{1}{2}gt^2 + V_0t + S_0$
 - a) Write a height equation. $S(t) = -4.9t^2 + 450$ m
 - b) Find the average velocity of the ball from t = 3 seconds to t = 5 seconds.
 - c) Find the velocity of the ball at 5 seconds.
 - d) When will the ball hit the ground?
 - e) With what velocity will the ball hit the ground?

b)
$$\Delta pos: = 327.5 - 405.9 = -39.2 \text{m/s}$$

 $\Delta time = 5-3$
(3, 405.9) (5, 327.5)

c)
$$v(t) = s'(t) = -9.8t$$

 $v(5) = -9.8(s) = -49 \text{ m/s}$

d)-4.9
$$t^2$$
+450=0
 $t=9.583$ sec