P Calculus AB

Notes – PVA

Motion along a Line – Suppose that an object is moving along a coordinate line (say the x-axis) so that we know its position s on that line as a function of time t: $s = f(t)$

The displacement of the object over the time interval from t to $t + \Delta t$ is $\Delta s = f(t+\Delta t) - f(t)$ and the **average velocity** of the object over that time interval is $\frac{\text{displacement}}{\text{travel time}} = \frac{\Delta s}{\Delta t} = \frac{f(t+\Delta t)-f(t)}{\Delta t}$

If we have a position function, the first derivative will give us velocity, and the second derivative will give us acceleration.

DEFINITION	SYMBOLIC RULE	
Instantaneous Velocity – is the derivative of the position function with respect to time.	$v(t) = \frac{ds}{dt} = \lim_{\Delta t \to 0} \frac{f(t+\Delta t)-f(t)}{\Delta t}$	The earliest questions that motivated the discovery of calculus were concerned with velocity & acceleration, particularly of freely falling bodies under the force of gravity. The mathematical description of this type of motion captured the imagination of many great scientists, including Aristotle, Galileo, and Newton. Experimental and theoretical investigations revealed that the distance a body released from rest falls freely is proportional to the square of the amount of time it has fallen. We express this by saying: $s = \frac{1}{2}gt^2$
Speed – is the absolute value of velocity.	speed $= \lvert v(t) \rvert = \left\lvert \frac{ds}{dt} \right\rvert$	
Acceleration – is the derivative of velocity with respect to time.	$a(t) = \frac{dv}{dt} = \frac{d^2s}{dt^2}$	**FREE FALL CONSTANTS (on Earth)** English units: $g = 32$ ft/sec^2 $s(t) = \frac{1}{2}(32)t^2 = 16t^2$ Metric units: $g = 9.8$ m/sec^2 $s(t) = \frac{1}{2}(9.8)t^2 = 4.9t^2$

English	**Calculus**
particle at rest	velocity = 0
particle moving right	velocity > 0
particle moving left	velocity < 0
particle changes direction	velocity changes sign
total distance traveled from time t_1 to t_2(where t_c = time when the particle changes direction)	$\lvert s(t_1)-s(t_c)\rvert + \lvert s(t_c)-s(t_2)\rvert$

Function	*Units*	*Examples / Abbreviations*
$s(t)$	linear units	feet or meters: ft or m
$v(t)$	linear units per unit of time	feet per second: ft/s meters per second: m/s
$a(t)$	linear units per unit of time squared	feet per second squared: ft/s^2 centimeters per second squared: cm/s^2

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- ∞ Particle moves forward when velocity is positive.
- ∞ Particle moves backward when velocity is negative.
- ∞ Particle standing still or at rest when graph of position is horizontal. (think – velocity is zero)
- ∞ Particle moving right when graph of position has a positive slope.
- ∞ Particle moving left when graph of position has a negative slope.
- ∞ Particle moves at its greatest speed when absolute value of velocity is maximized.
- ∞ Particle's acceleration is zero when velocity is constant.
- ∞ Particle's acceleration is negative when velocity is decreasing.
- ∞ Particle's acceleration is positive when velocity is increasing.
- ∞ Particle speeds up when velocity is negative and decreasing or velocity is positive and increasing. accel. & vel. have the same sign
- ∞ Particle slows down when velocity is positive and decreasing or velocity is negative and increasing. accel & vel have different signs

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VA class notes

1. The position function of a particle moving along the x-axis is given by $x(t) = t^3 - 12t^2 + 36t - 20$ for $0 \leq t \leq 8$.

   a) Find the velocity and acceleration of the particle.
   b) Find the open interval when the particle is moving to the left.
   c) Find the velocity of the particle when the acceleration is 0.
   d) Describe the motion of the particle.

a) $v(t) = x'(t) = 3t^2 - 24t + 36$

$a(t) = v'(t) = x''(t) = 6t - 24$

b) $3t^2 - 24t + 36 = 0$

$3(t^2 - 8t + 12) = 0$

$3(t-6)(t-2) = 0$

$t = 6,\ t = 2$

+ − +

2 6

moving to left ⇒ v(t) is neg

(2, 6)

c) $6t - 24 = 0$

$t = 4$

$v(4) = 3(4)^2 - 24(4) + 36 = -12$

d)

t	x(t)
0	-20
2	12
6	-20
8	12

The particle is 20 units to the left of the origin at t=0. The particle moves to the right 32 units until t=2. Between t=2 and 6 the particle moves to the left 32 units. Between t=6 and 8, the particle moves to the right 32 unit.

2. The position of a particle is given by the equation $s(t) = t^3 - 6t^2 + 9t$ where t is measured in seconds and s is measured in meters.

   a) Find the velocity at time t. $v(t) = s'(t) = 3t^2 - 12t + 9$ m/s
   b) What is the velocity at 2 s? at 4 s? $v(2) = 3(2)^2 - 12(2) + 9 = -3$ m/s
      $v(4) = 3(4)^2 - 12(4) + 9 = 9$ m/s
   c) When is the particle at rest?
   d) When is the particle moving to the right?
   e) Find the total distance traveled by the particle during the first 5 s.

c) $3t^2 - 12t + 9 = 0$

$3(t^2 - 4t + 3) = 0$

$3(t-3)(t-1) = 0$

$t = 3$ sec $t = 1$ sec

d) moving right ⇒ vel is pos.

+ − +

1 3

$(0,1)$ and $(3,\infty)$

e)

t	s(t)	
0	0	
		>4
1	4	
		>4
3	0	
		>20
5	20	

28m

3. Suppose that a ball is dropped from the upper observation deck of the CN Tower in Toronto, 450 m above the ground. $s(t) = -\frac{1}{2}gt^2 + v_0t + s_0$

   a) Write a height equation. $s(t) = -4.9t^2 + 450$ m
   b) Find the average velocity of the ball from t = 3 seconds to t = 5 seconds.
   c) Find the velocity of the ball at 5 seconds.
   d) When will the ball hit the ground?
   e) With what velocity will the ball hit the ground?

b) $\frac{\Delta \text{pos.}}{\Delta \text{time}} = \frac{327.5 - 405.9}{5 - 3} = -39.2$ m/s

$(3, 405.9)\ (5, 327.5)$

c) $v(t) = s'(t) = -9.8t$

$v(5) = -9.8(5) = -49$ m/s

d) $-4.9t^2 + 450 = 0$

$t = 9.583$ sec

e) $v(9.583)$

$= -93.915$ m/s