

ICM Notes on Rational Functions

A **rational function** has form $f(x) = \frac{p(x)}{q(x)}$ where $p(x)$ and $q(x)$ are polynomials.

examples of rational functions:

$$y = \frac{1}{x} \quad f(x) = \frac{x-2}{x+7} \quad g(x) = \frac{x^2+4x+5}{x^3}$$

The line $x = c$ is a **vertical asymptote** of the function f if as $x \rightarrow c$, $f(x) \rightarrow \pm\infty$.

- ❖ set the denominator = 0 and solve (be careful if the numerator and denominator have common factors which indicates a hole rather than an asymptote)

The line $y = c$ is a **horizontal asymptote** of the function f if as $x \rightarrow \pm\infty$, $f(x) \rightarrow c$.

- ❖ find the limit as x approaches $\pm\infty$

OR

compare the degree of the numerator with the degree of the denominator

"top heavy" means there is no horizontal asymptote

"bottom heavy" means $y = 0$ is a horizontal asymptote

"equal" means $y = \frac{\text{leading coefficient of numerator}}{\text{leading coefficient of denominator}}$ is a horizontal asymptote

A rational function has a **slant asymptote** when the degree of the numerator is one larger than the degree of the denominator.

- ❖ find by dividing the numerator by the denominator and ignoring the remainder

Example 1 Identify any asymptotes for each rational function.

A. $f(x) = \frac{x+3}{x^2-2x-8} = \frac{x+3}{(x-4)(x+2)}$ V.A. $x=4$
 $x=-2$ H.A. $y=0$

B. $g(x) = \frac{x+5}{x^2-25} = \frac{x+5}{(x-5)(x+5)} = \frac{1}{x-5}$ V.A. $x=5$ H.A. $y=0$
hole when $x=-5 \Rightarrow y = -\frac{1}{10}$

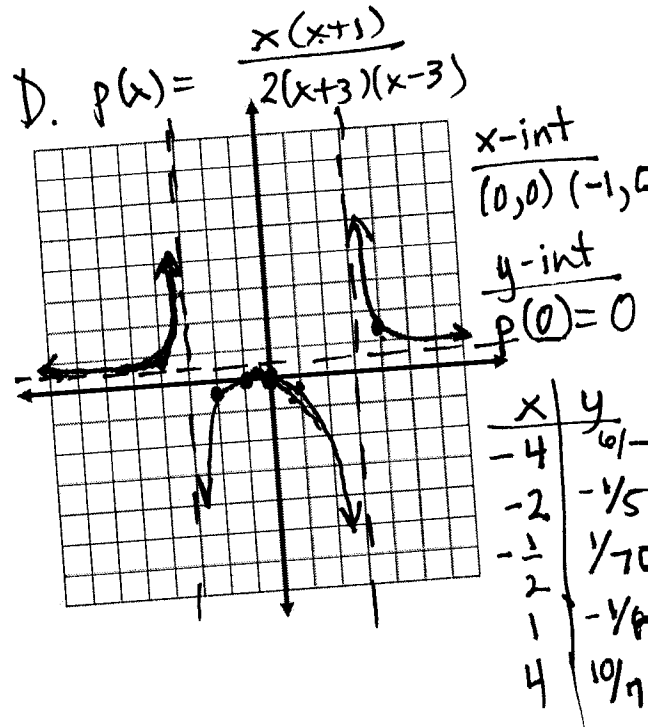
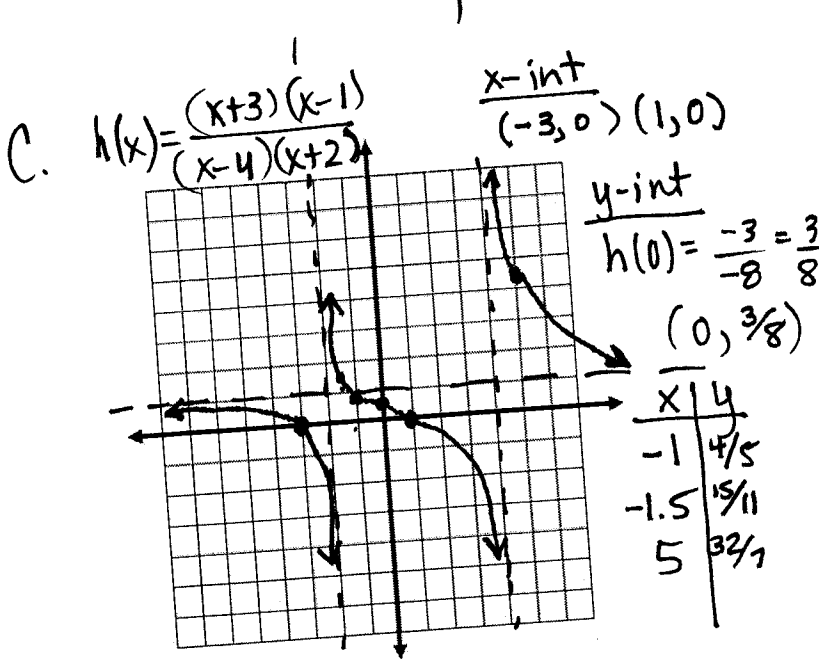
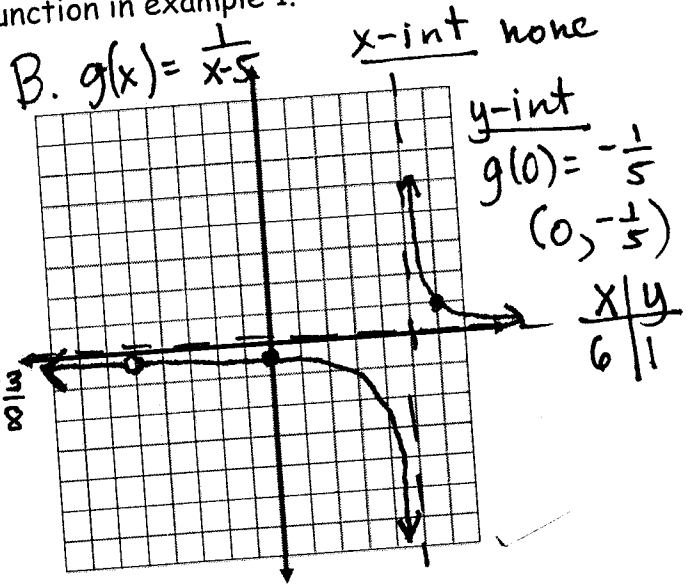
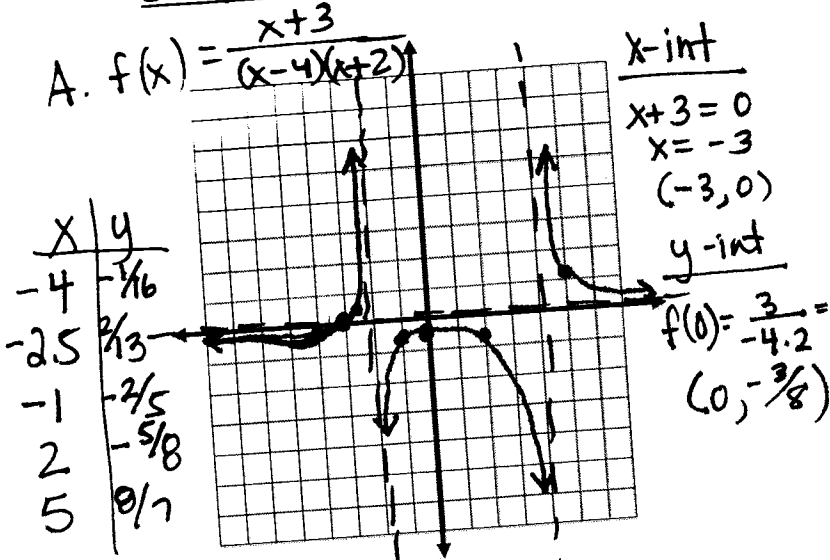
C. $h(x) = \frac{1x^2+2x-3}{1x^2-2x-8} = \frac{(x+3)(x-1)}{(x-4)(x+2)}$ V.A. $x=4$
 $x=-2$ H.A. $y=1$

D. $p(x) = \frac{1x^2+x}{2x^2-18} = \frac{x(x+1)}{2(x^2-9)} = \frac{x(x+1)}{2(x+3)(x-3)}$ V.A. $x=-3$
 $x=3$ H.A. $y = \frac{1}{2}$

Graphing a rational function

1. find and graph the x-intercept(s) Let $y=0$, solve for x .
2. find and graph the y-intercept Let $x=0$, solve for y .
3. find and graph any asymptotes
4. find and plot additional points as necessary to help fill in the graph

Example 2 Sketch a graph of each rational function in example 1.



Example 3 Find the slant asymptote: $f(x) = \frac{x^2 - 3x + 4}{x - 2}$

$$\begin{array}{r} 2 \overline{) 1 \ -3 \ 4} \\ \underline{2 \ -2} \\ 1 \ -1 \ 2 \\ \text{x const. rem} \end{array}$$

$y = x - 1$