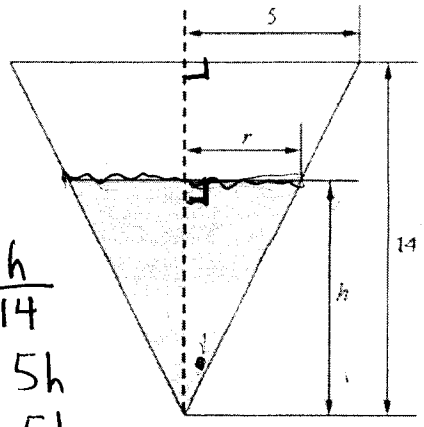
Ex 4) A conical tank of water is leaking water at a constant rate of 20 ft^3/hr. The base radius of the tank is 5 ft, and the height of the tank is 14ft.

a) At what rate is the depth of the water in the tank changing when the depth of the water is 6 ft?

b) At what rate is the radius of the top of the water changing when the depth of the water is 6 ft?



$\frac{r}{5} = \frac{h}{14}$

$14r = 5h$

$r = \frac{5h}{14}$

$\frac{dV}{dt} = -20\ ft^3/hr$

a) Find $\frac{dh}{dt}$ when $h = 6$ ft

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{5h}{14}\right)^2 h = \frac{1}{3}\pi \cdot \frac{25h^2}{196} \cdot h = \frac{25\pi}{588} h^3$$

$$\frac{dV}{dt} = \frac{25\pi}{588} \cdot 3h^2 \frac{dh}{dt}$$

$$-20 = \frac{25\pi}{588} \cdot 3(6)^2 \frac{dh}{dt}$$

$$-20 = \frac{225}{49}\pi \frac{dh}{dt}$$

$$\frac{-196}{45\pi} = \frac{dh}{dt}$$

when $h = 6$ ft, the height of the water is decreasing at a rate of $\frac{196}{45\pi}$ ft/hr

b) Find $\frac{dr}{dt}$ when $h = 6$ ft

$$r = \frac{5h}{14}$$

$$\frac{dr}{dt} = \frac{5}{14} \cdot \frac{dh}{dt} = \frac{5}{14}\left(\frac{-196}{45\pi}\right) = -\frac{14}{9\pi}$$

when $h = 6$ ft, the radius of the water is decreasing at a rate of $\frac{14}{9\pi}$ ft/hr