AP Calculus BC Review Notes Related Rates

You should be able to . . .

- Solve a related rates problem.

Related Rates Guidelines

1. Draw a picture and label the constants & variables.
2. Identify what you are asked to find.
3. Write an equation that relates the variables.
4. Differentiate with respect to t.
5. Interpret your answer and include units.

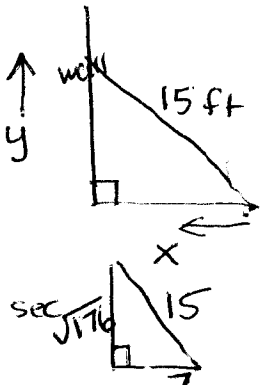
Ex 1) Air is being pumped into a spherical balloon at a rate of 5 cm^3/ min. Determine the rate at which the radius of the balloon is increasing when the diameter of the balloon is 20 cm.

$\frac{dV}{dt} = 5\ cm^3/min$ Find $\frac{dr}{dt}$ when $d = 20$ cm. → $r = 10$ cm

$V = \frac{4}{3}\pi r^3$ $5 = \frac{4}{3}\pi (10)^2 \cdot \frac{dr}{dt}$

$\frac{dV}{dt} = \frac{4}{3}\pi \cdot 3r^2 \frac{dr}{dt}$ $5 = 400\pi \cdot \frac{dr}{dt}$ $\frac{dr}{dt} = \frac{5}{400\pi} = \boxed{\frac{1}{80\pi}\ cm/min}$

Ex 2) A 15 ft ladder is resting against the wall. The bottom is initially 10 ft away from the wall and is being pushed towards the wall at a rate of $\frac{1}{4}$ ft/sec. How fast is the top of the ladder moving up the wall 12 seconds after we start pushing?



$\frac{dx}{dt} = -\frac{1}{4}\ ft/sec$

Find $\frac{dy}{dt}$ when $t = 12$ sec

$x^2 + y^2 = 15^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$2y\frac{dy}{dt} = -2x\frac{dx}{dt}$

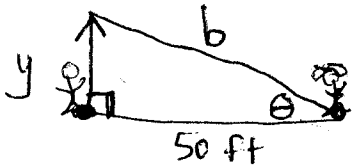
$\frac{2y}{2y}\frac{dy}{dt} = -\frac{x}{2y}\frac{dx}{dt}$

$\frac{dy}{dt} = -\frac{x}{y}\frac{dx}{dt}$

at 12 sec: $x = \sqrt{176}$, $y = 7$

$\frac{dy}{dt} = \frac{-(7)(-\frac{1}{4})}{\sqrt{176}} = \boxed{.132\ ft/sec}$

Ex 3) Two people are 50 ft apart. One of them starts walking north at a rate so that the angle is changing at a constant rate of 0.01 rad/min. At what rate is the distance between the two people changing when $\theta = 0.5$ radians?

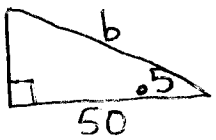


$\frac{d\theta}{dt} = 0.01\ rad/min$

Find $\frac{db}{dt}$ when $\theta = 0.5$ rad

$\cos(.5) = \frac{50}{b}$

$b = \frac{50}{\cos(.5)}$



$\cos\theta = \frac{50}{b} = 50b^{-1}$

$-\sin\theta \cdot \frac{d\theta}{dt} = -50b^{-2} \cdot \frac{db}{dt}$

$-\sin(.5)(.01) = \frac{-50}{(50.9)^2}\frac{db}{dt}$

$\boxed{\frac{db}{dt} = .311\ ft/min}$