AP Calculus BC Review   Notes Limits & Continuity

You should be able to . . .

- **Find the limit of a function using a table of values, a graph, or an algebraic technique.**

- **Apply L'Hopital's Rule:**
  Suppose that we have one of the following cases,

$$\lim_{x\to a}\frac{f(x)}{g(x)}=\frac{0}{0} \quad \text{OR} \quad \lim_{x\to a}\frac{f(x)}{g(x)}=\frac{\pm\infty}{\pm\infty}$$

where "a" can be any real number, infinity or negative infinity. In these cases we have,

$$\lim_{x\to a}\frac{f(x)}{g(x)}=\lim_{x\to a}\frac{f'(x)}{g'(x)}$$

- **Know the 2 special trigonometric limits:**

$$\boxed{\lim_{x\to 0}\frac{\sin x}{x}=1} \qquad \boxed{\lim_{x\to 0}\frac{1-\cos x}{x}=0}$$

- **Know/apply the definition of continuity at a point:**
  If $f(x)$ is continuous at $x=a$ then,

$$\lim_{x\to a^-}f(x)=f(a) \qquad \lim_{x\to a^+}f(x)=f(a) \qquad \lim_{x\to a}f(x)=f(a)$$

- **Determine if a function is continuous or discontinuous (holes, asymptotes, etc.)**

Ex 1) Find the limit numerically: $\lim_{x\to 0}\frac{1-\cos x}{x^2}=.5$ $\quad \frac{0}{0} \quad \lim_{x\to 0}\frac{\sin x}{2x} \quad \frac{0}{0} \quad \lim_{x\to 0}\frac{\cos x}{2}=\frac{1}{2}$

x	-0.1	-0.01	$-.001$	0	0.001	0.01	0.1
$g(x)$	.49958	.5	.5	error	.5	.5	.49958

Ex 2) Find the limit: $\lim_{x\to 0}\frac{x+\sin x}{x}$ $\quad \frac{0}{0} \quad \lim_{x\to 0}\frac{1+\cos x}{1}=\frac{1+1}{1}=2$

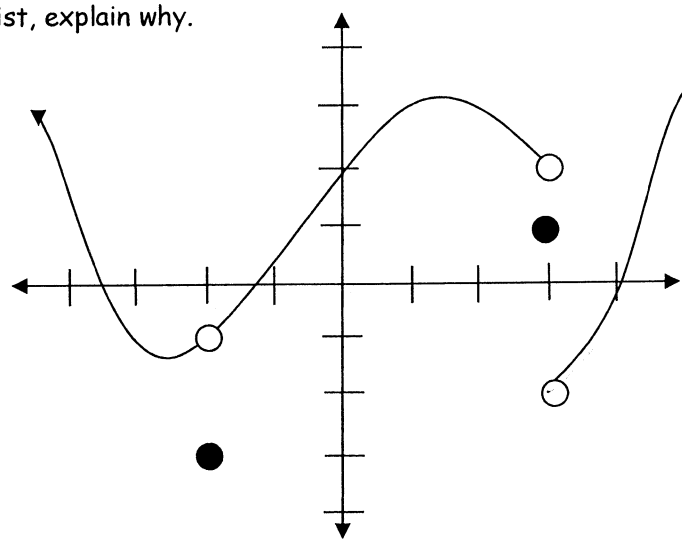
Ex 3) $f(x)=\begin{cases}x^2-1, & x<2\\ 3x-2, & x>2\end{cases}$

a) $\lim_{x\to 2^-}f(x)=2^2-1=3$ b) $\lim_{x\to 2^+}f(x)=3(2)-2=4$ c) $\lim_{x\to 2}f(x)=$ DNE

Ex 4) Find each limit:

a) $\lim_{x\to\infty}\frac{3x^3-x+1}{x+3} = \infty$ b) $\lim_{x\to\infty}\frac{4x^2-3x+5}{2x^3+x-1} = 0$ c) $\lim_{x\to\infty}\frac{3x+1}{x-4} = 3$

Ex 5) For the function f whose graph is given, state the value of the given quantity, if it exists. If it does not exist, explain why.



a) $\lim_{x\to 1} f(x) = 3$ b) $\lim_{x\to 3^-} f(x) = 2$ c) $\lim_{x\to 3^+} f(x) = -2$

d) $\lim_{x\to 3} f(x) =$ DNE e) $f(3) = 1$ f) $\lim_{x\to -2^-} f(x) = -1$

g) $\lim_{x\to -2^+} f(x) = -1$ h) $\lim_{x\to -2} f(x) = -1$ i) $f(-2) = -3$

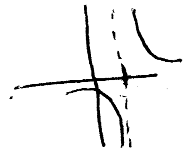
Ex 6) Given that $\lim_{x\to a} f(x) = -3$, $\lim_{x\to a} g(x) = 0$, $\lim_{x\to a} h(x) = 8$, find the limits that exist.

a) $\lim_{x\to a}[f(x)+h(x)] = -3+8 = 5$ b) $\lim_{x\to a}[f(x)]^2 = (-3)^2 = 9$

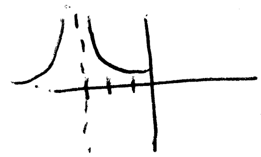
c) $\lim_{x\to a}\sqrt[3]{h(x)} = \sqrt[3]{8} = 2$ d) $\lim_{x\to a}\frac{1}{f(x)} = \frac{1}{-3}$

Ex 7) Find each limit:

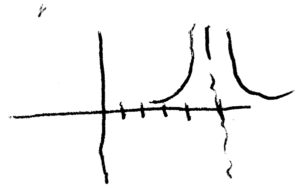
a) $\lim_{x\to 1^+} \frac{1}{x-1} = \infty$



b) $\lim_{x\to -3} \frac{1}{(x+3)^2} = \infty$



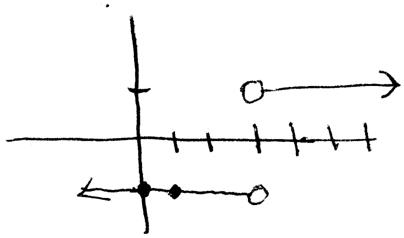
c) $\lim_{x\to 5^-} \frac{1}{(5-x)^2} = \infty$



d) $\lim_{x\to 2} \frac{\sin(x^2-4)}{x-2}$ $\quad \frac{0}{0} \quad \lim_{x\to 2} \frac{\cos(x^2-4)\cdot(2x)}{1} = \cos 0 \cdot 4 = \boxed{4}$

e) $\lim_{x\to\infty} \frac{\ln(x-4)}{x} = 0$

f) $\lim_{x\to 3} \frac{|x-3|}{x-3} =$ DNE



Ex 8) Given the graph of $f(x)$ below, determine if $f(x)$ is continuous at $x=-2$, $x=0$, and $x=3$.

no yes no