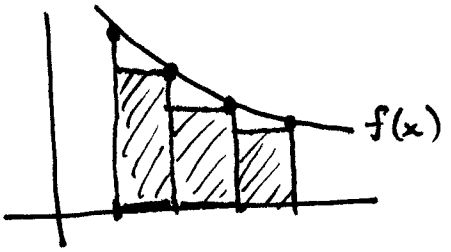
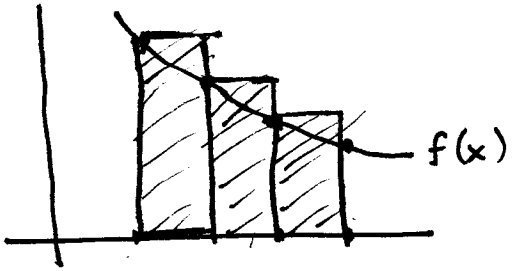
Area of a Region

inscribed rectangles – drawn below the curve



circumscribed rectangles – drawn above the curve



"lower sum" – underestimation – use inscribed rectangles

"upper sum" – overestimation – use circumscribed rectangles

Riemann sums

LRAM – use left endpts.

RRAM – use right endpts.

MRAM – use the midpt of each sub-interval

} to determine height of each rect.

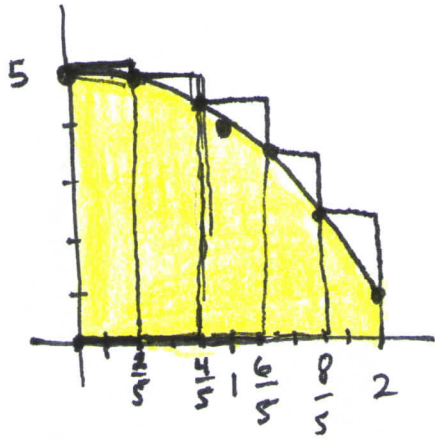
EX 1 $f(x) = 5 - x^2$ for $[0, 2]$

$n = 5$ subintervals

Find LRAM and RRAM.

$$\text{width} = \frac{\text{length of interval}}{\#\text{ subintervals}} = \frac{2-0}{5} = \frac{2}{5}$$

LRAM

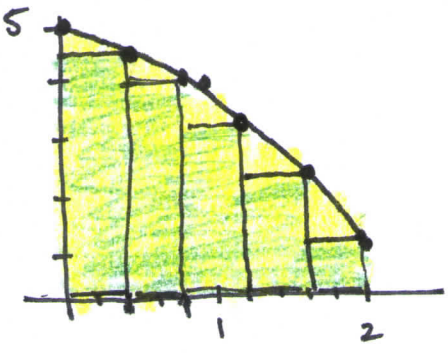


x	$f(x)$
0	5
$\frac{2}{5}$	$5 - \frac{4}{25} = \frac{121}{25}$
$\frac{4}{5}$	$5 - \frac{16}{25} = \frac{109}{25}$
$\frac{6}{5}$	$5 - \frac{36}{25} = \frac{89}{25}$
$\frac{8}{5}$	$5 - \frac{64}{25} = \frac{61}{25}$

$$\frac{2}{5}\left(5 + \frac{121}{25} + \frac{109}{25} + \frac{89}{25} + \frac{61}{25}\right)$$

$$= 8.08$$

RRAM



x	$f(x)$
$\frac{2}{5}$	$\frac{121}{25}$
$\frac{4}{5}$	$\frac{109}{25}$
$\frac{6}{5}$	$\frac{89}{25}$
$\frac{8}{5}$	$\frac{61}{25}$
2	1

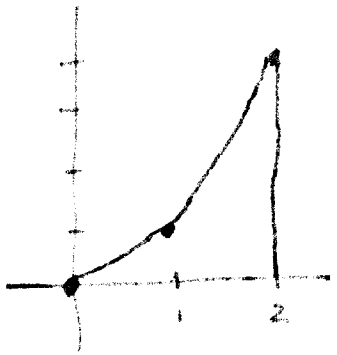
$$\frac{2}{5}\left(\frac{121}{25} + \frac{109}{25} + \frac{89}{25} + \frac{61}{25} + 1\right)$$

$$= 6.48$$

EX 2 $f(x) = x^2$ for $[0, 2]$

$n = 8$ subintervals

Find LRAM, RRAM, & MRAM.



subinterval width $= \frac{2-0}{8} = \frac{1}{4}$

x	$f(x)$
0	0
1/4	1/16
1/2	1/4
3/4	9/16
1	1
5/4	25/16
3/2	9/4
7/4	49/16
2	4
1/8	1/64
3/8	9/64
5/8	25/64
7/8	49/64
9/8	81/64
11/8	121/64
13/8	169/64
15/8	225/64

$$LRAM = \frac{1}{4}\left(0 + \frac{1}{16} + \frac{1}{4} + \frac{9}{16} + 1 + \frac{25}{16} + \frac{9}{4} + \frac{49}{16}\right)$$

$$= 2.1875$$

$$RRAM = \frac{1}{4}\left(\frac{1}{16} + \frac{1}{4} + \frac{9}{16} + 1 + \frac{25}{16} + \frac{9}{4} + \frac{49}{16} + 4\right)$$

$$= 3.1875$$

$$MRAM = \frac{1}{4}\left(\frac{1}{64} + \frac{9}{64} + \frac{25}{64} + \frac{49}{64} + \frac{81}{64} + \frac{121}{64} + \frac{169}{64} + \frac{225}{64}\right)$$

$$= 2.65625$$

EX 3

		2	1	4	2	
x	0	2	3	7	9	
f(x)	3	6	7	6	8	

Find the approximate area using LRAM and RRAM.

LRAM $2(3) + 1(6) + 4(7) + 2(6) = 52$

RRAM $2(6) + 1(7) + 4(6) + 2(8) = 59$