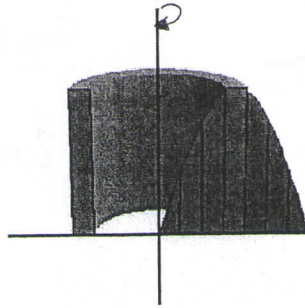# Notes ---- Volume: Shell Method

- The rectangle is __parallel__ to the axis of __revolution__ and __perpendicular__ to the axis of __integration__.

Generation of Typical Shell

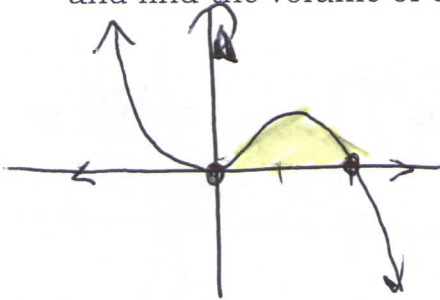- Formula is $V = 2\pi \int_a^b x f(x)\, dx$

Why is this method helpful?

$x = 0, 2$
$x^2(2-x) = 0$

- Suppose you have the region: $y = 2x^2 - x^3$ and $y = 0$. Revolve around the y-axis and find the volume of the solid. What is the issue?

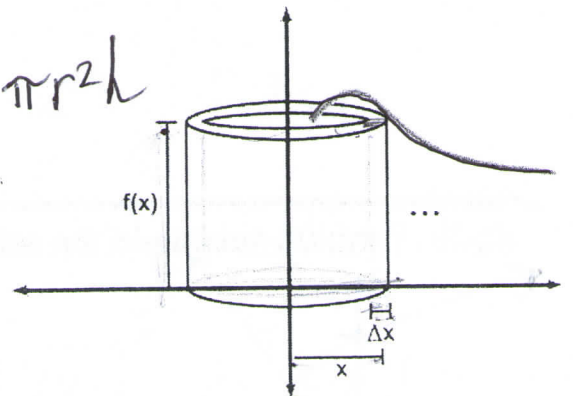Can't solve for x

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Deriving the Shell Method Formula

outer radius $= r_2$
inner radius $= r_1$
thickness of shell $= r_2 - r_1$
average radius $= \dfrac{r_1 + r_2}{2} = $ "r"

$V_{cyl} = \pi r^2 h$

f(x)

Δx

x

$V = \pi (r_2)^2 h - \pi (r_1)^2 h$

$= \pi h \left((r_2)^2 - (r_1)^2\right)$

$= \pi h (r_2 - r_1)(r_2 + r_1) \cdot 2$ avg radius $= r$

$\underbrace{\dfrac{}{2}}$

$= 2\pi r h (r_2 - r_1)$

radius   height   thickness

$\boxed{V = 2\pi \int_a^b \underset{rad.}{x} \cdot \underset{ht}{f(x)}\, \underset{thickness}{dx}}$

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Ex1) Find the volume of the solid: region is $y = x - x^3$, $y = 0$, from $x = 0$ to $x = 1$; revolve around the y-axis.
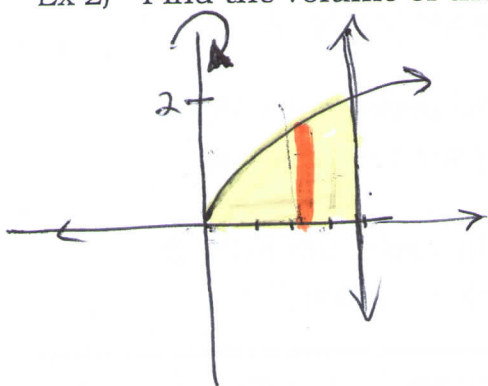
$$x(1-x^2) = 0$$
$$x(1-x)(1+x) = 0$$
$$x = 0, 1, -1$$
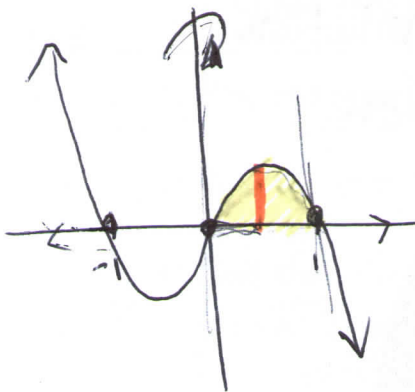
Shell

$$V = 2\pi \int_0^1 x(x-x^3)\,dx = 2\pi \cdot \frac{2}{15} = \boxed{\frac{4\pi}{15}}$$

Ex 2) Find the volume of the solid: region is $y = \sqrt{x}$, $y = 0$, $x = 4$; revolve around the y-axis.
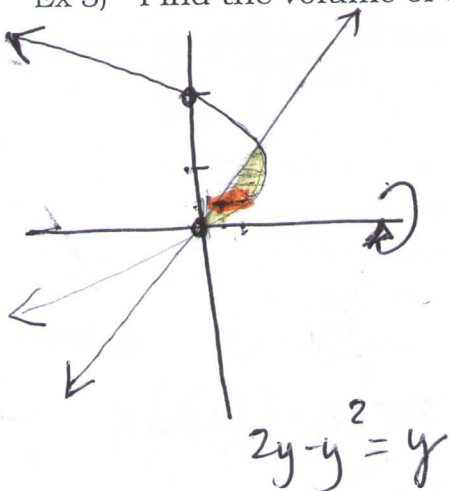
$$x = y^2$$

washer

$$\pi \int_0^2 \left[(4)^2 - (y^2)^2\right]\,dy = \boxed{\frac{128}{5}\pi}$$

shell

$$2\pi \int_0^4 x(\sqrt{x})\,dx = \boxed{80.425}$$

Ex 3) Find the volume of the solid: region is $x = 2y - y^2$, $x = y$; revolve around the x-axis.

$$0 = 2y - y^2$$
$$0 = y(2-y)$$
$$y = 0, \ y = 2$$

Shell

$$2\pi \int_0^1 y(2y - y^2 - y)\,dy$$

$$= 2\pi \cdot \frac{1}{12} = \boxed{\frac{\pi}{6}}$$

$$2y - y^2 = y$$
$$0 = y^2 - y$$
$$0 = y(y-1) \quad y = 0, y = 1$$

$$x = \sqrt{y}$$

Ex 4) Find the volume of the solid: region is $y = x^2$, $x = 2$, $y = 0$; revolve around $x = 2$.
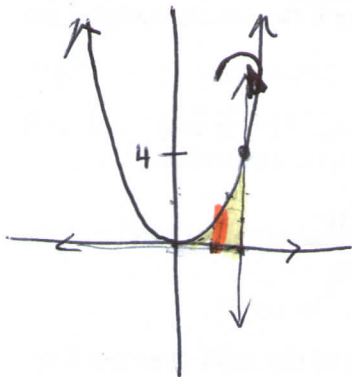
Disk $\quad \pi \displaystyle\int_0^4 \left(2 - \sqrt{y}\right)^2 dy = \boxed{\dfrac{8\pi}{3}}$

Shell $\quad 2\pi \displaystyle\int_0^2 (2-x) x^2 \, dx = 2\pi \cdot \dfrac{4}{3} = \boxed{\dfrac{8\pi}{3}}$

Ex 5) Find the volume of the solid: region is $y = x - x^2$, $y = 0$; revolve around $x = 3$.
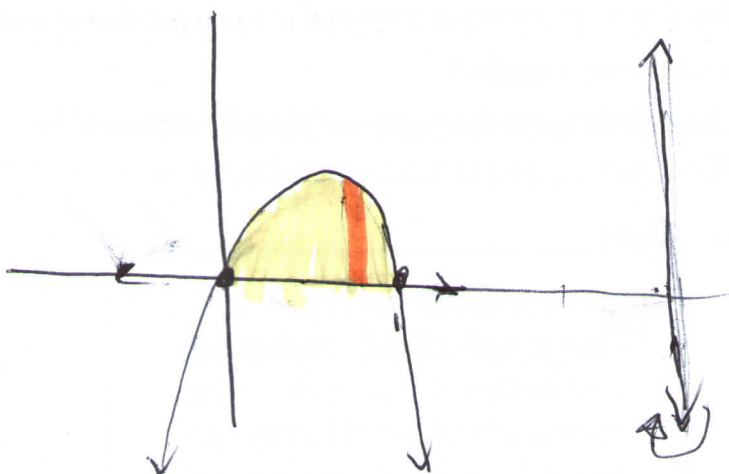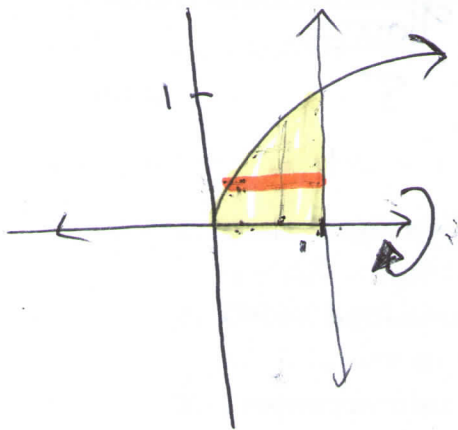
$x(1-x) = 0$
$x = 0, 1$

Shell
$2\pi \displaystyle\int_0^1 (3-x)(x - x^2) \, dx$

$2\pi \cdot \dfrac{5}{12} = \boxed{\dfrac{5\pi}{6}}$

Ex 6) Find the volume of the solid: region is $y = \sqrt{x}$, $y = 0$, $x = 1$; revolve around the x-axis

$x = y^2$

disk $\quad \pi \displaystyle\int_0^1 \left(\sqrt{x}\right)^2 dx = \boxed{\dfrac{\pi}{2}}$

Shell
$2\pi \displaystyle\int_0^1 y \left(1 - y^2\right) dy = \boxed{\dfrac{\pi}{2}}$

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