

Review Unit #1: Limits & Continuity

Multiple-Choice Part 1: Non-Calculator (22 minutes)

1) Given constants $a, b, & c$, and $a < b < c$, and function f is continuous on the closed interval $[a, c]$ which of the following must be equal?

- I. $\lim_{x \rightarrow b^-} f(x)$ II. $\lim_{x \rightarrow b^+} f(x)$ III. $f(b)$

- (A) I & II only (B) II & III only (C) I, & III only
 (D) all of them must be equal (E) none of them must be equal.

2) If $f(x) = \begin{cases} \ln x & \text{for } 0 < x \leq 2 \\ x^2 \ln 2 & \text{for } 2 < x \leq 4, \end{cases}$ then $\lim_{x \rightarrow 2} f(x) = \ln 2 \neq 2^2 \ln 2$

- (A) $\ln 2$ (B) $\ln 8$ (C) $\ln 16$ (D) 4 (E) nonexistent

3) $\lim_{x \rightarrow e} \frac{x}{\ln x} = \frac{e}{\ln e} = \frac{e}{1} = e$

- (A) 0 (B) $\frac{1}{e}$ (C) 1 (D) e (E) nonexistent

4) Which of the following functions are continuous for all real numbers x ?

- I. $y = x^{2/3}$ II. $y = e^x$ III. $y = \tan x$ ← has asymptotes

- (A) None (B) I only (C) II only (D) I & II (E) I & III

5) If $\begin{cases} f(x) = \frac{\sqrt{2x+5} - \sqrt{x+7}}{x-2}, & \text{for } x \neq 2, \\ f(2) = k \end{cases}$ and if f is continuous at $x = 2$, then $k =$

$\lim_{x \rightarrow 2} \left(\frac{\sqrt{2x+5} - \sqrt{x+7}}{x-2} \right) \left(\frac{\sqrt{2x+5} + \sqrt{x+7}}{\sqrt{2x+5} + \sqrt{x+7}} \right)$
 $= \lim_{x \rightarrow 2} \frac{2x+5 - (x+7)}{(x-2)(\sqrt{2x+5} + \sqrt{x+7})} = \lim_{x \rightarrow 2} \frac{1}{\sqrt{2x+5} + \sqrt{x+7}}$
 $= \frac{1}{\sqrt{9} + \sqrt{9}} = \frac{1}{6}$

- (A) 0 (B) $\frac{1}{6}$ (C) $\frac{1}{3}$ (D) 1 (E) $\frac{7}{5}$

6) $\lim_{n \rightarrow \infty} \frac{4n^2}{n^2 + 10,000n} =$

- (A) 0 (B) $\frac{1}{2,500}$ (C) 1 (D) 4 (E) nonexistent

7) $\lim_{x \rightarrow 0} (x \csc x)$ is $= \lim_{x \rightarrow 0} \frac{x}{\sin x}$

- (A) $-\infty$ (B) -1 (C) 0 (D) 1 (E) ∞

8) $\lim_{x \rightarrow 4} \frac{x^2 - 5x - 12}{x^2 - 8}$ is $\frac{16 - 20 - 12}{16 - 8} = -2$

- (A) $-\infty$ (B) -2 (C) 2 (D) 1 (E) ∞

9) $\lim_{x \rightarrow 0} \frac{3 \cos x - 3}{x}$ is $\lim_{x \rightarrow 0} \frac{3(\cos x - 1)}{x} = -3 \cdot 0 = 0$

- (A) $-\infty$ (B) -3 (C) 0 (D) 3 (E) ∞

$\frac{3(-1)(1 - \cos x)}{x}$

10) For which function below is $\lim_{x \rightarrow \infty} f(x) = 0$ is

- i. $f(x) = \frac{-x}{3^x}$ ii. $f(x) = x(3^x)$ iii. $f(x) = \frac{x}{\log_3 x}$ iv. $f(x) = \frac{3^x}{x}$

- (A) ii only (B) iv only (C) ii & iv only (D) all of these (E) none of these

11) $\lim_{x \rightarrow \infty} \frac{5 + \sqrt[3]{x}}{\sqrt[3]{x} - 4}$

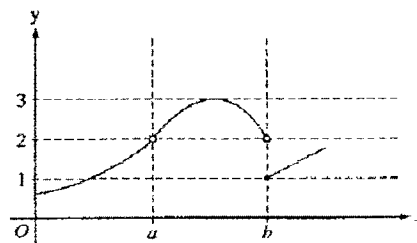
- (A) $-\infty$ (B) -1 (C) 0 (D) 1 (E) $-\frac{5}{4}$

Multiple-Choice Part 2: Calculator (6 minutes)

1) If $a \neq 0$, then $\lim_{x \rightarrow a} \frac{x^2 - a^2}{x^4 - a^4}$ is $\frac{(x-a)(x+a)}{(x+a)(x-a)(x^2+a^2)} = \frac{1}{a^2+a^2} = \frac{1}{2a^2}$

- (A) $\frac{1}{a^2}$ (B) $\frac{1}{2a^2}$ (C) $\frac{1}{6a^2}$ (D) 0 (E) nonexistent

2) The graph of the function f is shown in the figure to the right. Which of the following statements about f is true?



- (A) $\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow b} f(x) = 2$ (B) $\lim_{x \rightarrow a} f(x) = 2$ (C) $\lim_{x \rightarrow b} f(x) = 2$ (D) $\lim_{x \rightarrow b} f(x) = 1$ (E) $\lim_{x \rightarrow a} f(x)$ does not exist.

3) The $\lim_{p \rightarrow \infty} \left(\frac{7}{e^p} - 1 \right) = 0 - 1 = -1$

- (A) 0 (B) 7 (C) ∞ (D) -1 (E) nonexistent

Free-Response Part 1: Non-Calculator (15 minutes)

The f be the function given by $f(x) = \frac{x}{\sqrt{x^2 - 4}}$

- (a) Find the domain of f . $(x-2)(x+2) > 0$ $\frac{+}{-2} \frac{-}{2} \frac{+}{2}$ $(-\infty, -2) \cup (2, \infty)$
- (b) Write an equation for each vertical asymptote to the graph of f .
 $x = 2$ $x = -2$ (where denom. is undefined)
- (c) Write an equation for each horizontal asymptote of the graph of f .
 $y = 1$ $y = -1$ ($\lim_{x \rightarrow \pm \infty} \frac{x}{\sqrt{x^2 - 4}} = \pm 1$)
- (d) Find the one-sided limits of $f(x)$ as $x \rightarrow 2$ and as $x \rightarrow -2$.

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x	$f(x)$	$\lim_{x \rightarrow 2^+} f(x) = \infty$	$\lim_{x \rightarrow -2^+} f(x) = \text{DNE}$
4	$\frac{4}{\sqrt{2}}$	$\lim_{x \rightarrow 2^-} f(x) = \text{DNE}$	$\lim_{x \rightarrow -2^-} f(x) = -\infty$
3	$\frac{3}{\sqrt{5}}$		

x	$f(x)$
-4	$-\frac{4}{\sqrt{2}}$
-3	$-\frac{3}{\sqrt{5}}$