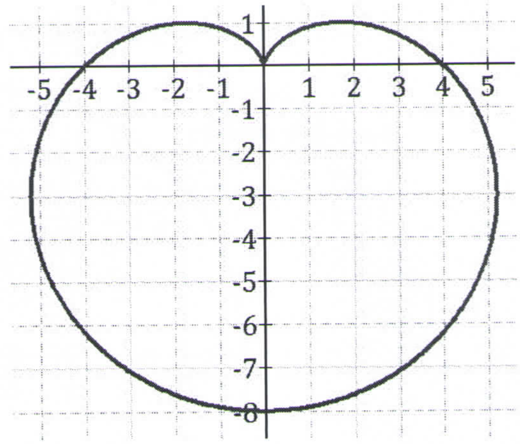
The graph of the polar curve $r = 4 - 4\sin\theta$ is shown to the right.



(You may use your calculator for all sections of this problem.)

a) For $0 \le \theta < 2\pi$, there are two points on r with y-coordinate equal to -4. Find the subject points. Express your answers using polar coordinates.

$$y = r \cdot \sin\theta = (4 - 4\sin\theta)\sin\theta = -4$$
$$\rightarrow \theta \approx 3.8078 \quad \text{and} \quad 5.6169$$
$$\theta \approx 3.8078 \rightarrow r = 4 - 4\sin 3.8078 = 6.472$$
$$\rightarrow (6.472, 3.8078)$$
$$\theta \approx 5.6169 \rightarrow r = 4 - 4\sin 5.6169 = 6.472$$
$$\rightarrow (6.472, 5.6169)$$

b) Write an expression for the x-coordinate of each point on the graph of $r = 4 - 4\sin\theta$. Express your answer in terms of θ.

$$x = r \cdot \cos\theta = (4 - 4\sin\theta)\cos\theta$$

c) A particle moves along the polar curve $r = 4 - 4\sin\theta$ so that at time t seconds, $\theta = t^2$. Find the time t in the time interval $1 \le t \le 2$ for which the x-coordinate of the particle's position is -1.

$$x = (4 - 4\sin t^2)\cos t^2 = -1 \rightarrow t \approx 1.5536$$

d) Find $\left.\dfrac{dr}{dt}\right|_{t=2}$. Interpret the meaning of your answer in the context of the problem.

$$r = 4 - 4\sin t^2$$
$$\text{By hand: } \frac{dr}{dt} = -8t\cos t^2 \rightarrow \left.\frac{dr}{dt}\right|_{t=2} = -16\cos 4$$
$$\text{Using a calculator: } \left.\frac{d}{dt}(4 - 4\sin t^2)\right|_{t=2} \approx 10.458$$

As the particle moves on the graph of $r = 4 - 4\sin\theta$, when $t = 2$ seconds the distance to the pole is increasing at a rate equal to 10.458 units per second.

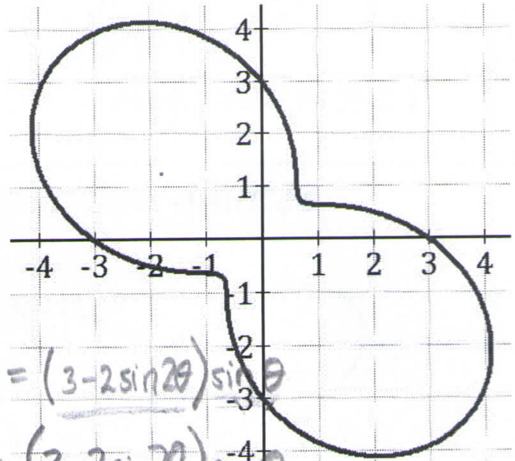
e) Find $\left.\dfrac{dx}{dt}\right|_{t=2}$. Interpret the meaning of your answer in the context of the problem.

$$\text{Using a calculator: } \left.\frac{d}{dt}((4 - 4\sin t^2)\cos t^2)\right|_{t=2} \approx 14.4368$$

As the particle moves on the graph of $r = 4 - 4\sin\theta$, when $t = 2$ seconds the particle moves to the right with a horizontal speed equal to 14.4368 units per second.

The graph of the polar curve $r = 3 - 2\sin(2\theta)$ for $0 \le \theta < 2\pi$ is shown to the right.

(You may use your calculator for all sections of this problem.)



$y = (3 - 2\sin 2\theta)\sin\theta$

$x = (3 - 2\sin 2\theta)\cos\theta$

a) Write in terms of θ an expression for $\frac{dy}{dx}$, the slope of the tangent line to the graph of r.

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{-4\cos 2\theta \sin\theta + (3 - 2\sin(2\theta))\cos\theta}{-4\cos 2\theta \cos\theta - (3 - 2\sin(2\theta))\sin\theta}$$

b) Find the coordinates of the point where curve r has a vertical tangent line in the interval $0 \le \theta < \pi$. Write your answer using polar coordinates.

$$\frac{dy}{dx} \text{ is undefined} \rightarrow -4\cos 2\theta \cos\theta - (3 - 2\sin(2\theta))\sin\theta = 0 \rightarrow \theta \approx 2.670$$

$$r = 3 - 2\sin\big(2(2.670)\big) = 4.6177 \rightarrow (4.6177, 2.670)$$

c) Write in terms of x and y an equation for the line tangent to the graph of the curve r at the point where $\theta = \frac{\pi}{6}$.

$$\left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{6}} \approx -0.041 \quad \left(\text{by hand: } \left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{6}} = \frac{3\sqrt{3} - 5}{-3 - \sqrt{3}}\right)$$

$$\left.\begin{aligned} x &= r \cdot \cos\theta = \frac{3\sqrt{3} - 3}{2} = 1.098 \\ y &= r \cdot \sin\theta = \frac{3 - \sqrt{3}}{2} = 0.6339 \end{aligned}\right\} \rightarrow y - 0.6339 = -0.041(x - 1.098)$$

d) A particle moves along the polar curve $r = 3 - 2\sin(2\theta)$ so that $\frac{d\theta}{dt} = 2$ for all times $t \ge 0$.

Find the value of $\frac{dr}{dt}$ at $\theta = \frac{\pi}{6}$. Interpret the meaning of your answer in the context of the problem.

$$\frac{dr}{dt} = \frac{\frac{dr}{d\theta}}{\cancel{\frac{dt}{d\theta}}} =$$

$$\frac{dr}{dt} = \frac{dr}{d\theta} \cdot \frac{d\theta}{dt} = (-4\cos(2\theta))(2) = -8\cos(2\theta)$$

$$\left.\frac{dr}{dt}\right|_{\theta=\frac{\pi}{6}} = -4$$

As the particle moves on the graph of $r = 3 - 2\sin(2\theta)$, when it is at the point where $\theta = \frac{\pi}{6}$ radians the distance to the pole is decreasing at a rate equal to 4 units per second.

e) Assume now that for the particle whose motion was described in section (d) we have $\theta = 2t$. Find the position vector of the particle $\langle x(t), y(t)\rangle$ in terms of t. Use your calculator to find the velocity vector and the speed of the particle at $t = 1.5$.

$$\left.\begin{aligned} x &= r \cdot \cos\theta = (3 - 2\sin(4t))\cos(2t) \\ y &= r \cdot \sin\theta = (3 - 2\sin(4t))\sin(2t) \end{aligned}\right\} \rightarrow \langle (3 - 2\sin(4t))\cos(2t), (3 - 2\sin(4t))\sin(2t)\rangle$$

$$\text{Velocity vector: } \left\langle \left.\frac{dx}{dt}\right|_{t=1.5}, \left.\frac{dy}{dt}\right|_{t=1.5} \right\rangle = \langle 6.600, -8.130\rangle$$

$$\text{Speed: } \sqrt{\left(\left.\frac{dx}{dt}\right|_{t=1.5}\right)^2 + \left(\left.\frac{dy}{dt}\right|_{t=1.5}\right)^2} = 10.472$$