Integration Review—Riemann and Trapezoidal Sums

1. A tank contains 120 gallons of oil at time $t = 0$ hours. Oil is being pumped into the tank at a rate $R(t)$, where $R(t)$ is measured in gallons per hour and t is measured in hours. Selected values of $R(t)$ are given in the table below.

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t (hours)	0	3	5	9	12
$R(t)$ (gallons per hour)	8.9	6.8	6.4	5.9	5.7

(a) Estimate the number of gallons of oil in the tank at $t = 12$ hours by using a trapezoidal approximation with four subintervals and values from the table. Show the computations that lead to your answer.

(b) A model for the rate at which oil is being pumped into the tank is given by the function $G(t) = 3 + \frac{10}{1+\ln(t+2)}$, where $G(t)$ is is measured in gallons per hour and t is measured in hours. Use the model to find the number of gallons of oil in the tank at $t = 12$ hours.

a) amt added $= \frac{1}{2}(3)(8.9+6.8) + \frac{1}{2}(2)(6.8+6.4) + \frac{1}{2}(4)(6.4+5.9) + \frac{1}{2}(3)(5.9+5.7$

$= 78.75$

total in tank at 12 hrs $= 120 + 78.75 = 198.75$ gal

b) $\int_0^{12} G(t)\,dt = A(12) - A(0)$

$A(0) + \int_0^{12} G(t)\,dt = A(12)$

$120 + 77.975 = 197.975$ gal

2. The temperature, in degrees Celsius (°C), of the water in a pond is a differentiable function W of time t. The table below shows the water temperature as recorded every 3 days over a 15-day period.

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t (days)	0	3	6	9	12	15
$W(t)$ (°C)	20	31	28	24	22	21

(a) Approximate the average temperature, in degrees Celsius, of the water over the time interval $0 \le t \le 15$ days by using a trapezoidal approximation with subintervals of length $\Delta t = 3$ days and values from the table. Show the computations that lead to your answer.

(b) A student proposes the function P, given by $P(t) = 20 + 10te^{(-t/3)}$, as a model for the temperature of the water in the pond at time t, where t is measured in days and $P(t)$ is measured in degrees Celsius. Use the function P to find the average value, in degrees Celsius, of $P(t)$ over the time interval $0 \le t \le 15$ days.

a) avg. temp $= \frac{1}{15-0}\left(\frac{1}{2}(3)\left[20 + 2\cdot31 + 2\cdot28 + 2\cdot24 + 2\cdot22 + 21\right]\right)$

$= \frac{1}{15}\left(\frac{3}{2}\cdot 251\right) = 25.1$°C

b) $\frac{1}{15}\int_0^{15}(20 + 10te^{-t/3})\,dt = 25.757$ °C

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3. A test plane flies in a straight line with positive velocity $v(t)$, in miles per minute at time t minutes, where v is a differentiable function of t. Selected values of $v(t)$ for $0 \le v(t) \le 40$ are shown in the table below.

t (min)	0	5	10	15	20	25	30	35	40
$v(t)$ (mpm)	7.0	9.2	9.5	7.0	4.5	2.4	2.4	4.3	7.2

(a) Use a midpoint Riemann sum with four subintervals of equal length and values from the table to approximate $\int_0^{40} v(t)\,dt$. Show the computations that lead to your answer. Using correct units, explain the meaning of $\int_0^{40} v(t)\,dt$ in terms of the plane's flight.

(b) The function f defined by $f(t) = 6 + \cos\left(\frac{t}{10}\right) + 3\sin\left(\frac{7t}{40}\right)$, is used to model the velocity of the plane, in miles per minute, for $0 \le t \le 40$. According to this model, what is the average velocity of the plane, in miles per minute over the time interval $0 \le v(t) \le 40$?

a) $\int_0^{40} v(t)dt \approx 10\left[9.2 + 7.0 + 2.4 + 4.3\right] = 229$ miles

Between $t=0$ min and $t=40$ min, the plane flies a total distance of 229 miles.

b) $\frac{1}{40-0}\int_0^{40}\left(6+\cos\left(\frac{t}{10}\right)+3\sin\left(\frac{7t}{40}\right)\right)dt = 5.916$ miles/min

4. Rocket A has positive velocity $v(t)$ after being launched upward from an initial height of 0 feet at time $t = 0$ seconds. The velocity of the rocket is recorded for selected values of t over the interval $0 \le t \le 80$ seconds, as shown in the table below.

t (seconds)	0	10	20	30	40	50	60	70	80
$v(t)$ (ft per second)	5	14	22	29	35	40	44	47	49

(a) Using correct units, explain the meaning of $\int_{10}^{70} v(t)\,dt$ in terms of the rocket's flight. Use a midpoint Riemann sum with 3 subintervals of equal length to approximate $\int_{10}^{70} v(t)\,dt$.

(b) Rocket B is launched upward with an acceleration of $a(t) = \frac{3}{\sqrt{t+1}}$ feet per second per second. At time $t = 0$ seconds, the initial height of the rocket is 0 feet, and the initial velocity is 2 feet per second. Which of the two rockets is traveling faster at $t = 80$ seconds? Explain your answer.

a) The distace traveled by the rocket in feet from $t=10$ sec to $t=70$ sec.

$\int_0^{70} v(t)dt \approx 20\left[22+35+44\right] = 2020$ feet

b) vel of rocket B $= \int \frac{3}{\sqrt{t+1}}dt$

$u = t+1$

$\frac{du}{dt} = 1 \rightarrow du = dt$

$3\int \frac{du}{\sqrt{u}} = 3\int u^{-1/2}du = 6u^{1/2} + C = 6\sqrt{t+1} + C$

use $(0,2)$: $2 = 6\sqrt{0+1} + C \rightarrow C = -4$

Vel of B $= 6\sqrt{t+1} - 4\Big|_{t=80\text{ sec}}$

$= 6\sqrt{81} - 4 = 50$ ft/sec

Vel of A is 49 ft/sec when $t = 80$ sec

rocket B is traveling faster since $50 > 49$

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