Taylor Series

Suppose $f(x)$ is infinitely differentiable at $x=c$.
The power series centered at c is

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(c)\cdot(x-c)^n}{n!} = f(c) + \frac{f'(c)}{1!}(x-c)^1 + \frac{f''(c)}{2!}(x-c)^2 + \dots$$

$$\frac{f^{(n)}(c)}{n!}(x-c)^n + \dots$$

EX1 $f(x) = e^{3x}$. Find the Maclaurin series.

$$\underbrace{\text{Maclaurin}}_{c=0}$$

		at $c=0$
$f(x)$	e^{3x}	1
$f'(x)$	$3e^{3x}$	3
$f''(x)$	$9e^{3x}$	9
$f'''(x)$	$27e^{3x}$	27

$$\frac{1}{0!} + \frac{3}{1!}x^1 + \frac{9}{2!}x^2 + \frac{27}{3!}x^3 + \dots$$

$$\sum_{n=0}^{\infty} \frac{3^n}{n!}x^n = \sum_{n=0}^{\infty} \frac{(3x)^n}{n!}$$

$$e^x = \sum \frac{1}{n!}x^n$$

$$e^{3x} = \sum \frac{3^n}{n!}x^n = \sum \frac{(3x)^n}{n!}$$

$$e^{4x} = \sum \frac{4^n}{n!}x^n = \sum \frac{(4x)^n}{n!}$$

EX2 $f(x) = \cos\sqrt{x}$. Find the Maclaurin series.

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} \qquad \cos\sqrt{x} = \sum \frac{(-1)^n(\sqrt{x})^{2n}}{(2n)!}$$

$$= \sum \frac{(-1)^n \left((\sqrt{x})^2\right)^n}{(2n)!}$$

$$= \sum \frac{(-1)^n x^n}{(2n)!}$$

EX 3 $f(x) = \sin x$. Find the Taylor series centered at $x = \pi/4$.

		$c = \pi/4$
$f(x)$	$\sin x$	$\sqrt{2}/2$
$f'(x)$	$\cos x$	$\sqrt{2}/2$
$f''(x)$	$-\sin x$	$-\sqrt{2}/2$
$f'''(x)$	$-\cos x$	$-\sqrt{2}/2$
$f^{(4)}(x)$	$\sin x$	$\sqrt{2}/2$

$$\frac{\frac{\sqrt{2}}{2}}{0!} + \frac{\frac{\sqrt{2}}{2}}{1!}\left(x - \frac{\pi}{4}\right)^1 + \frac{-\frac{\sqrt{2}}{2}}{2!}\left(x - \frac{\pi}{4}\right)^2 + \frac{-\frac{\sqrt{2}}{2}}{3!}\left(x - \frac{\pi}{4}\right)^3$$

$$+ \frac{\frac{\sqrt{2}}{2}}{4!}\left(x - \frac{\pi}{4}\right)^4 + \frac{\frac{\sqrt{2}}{2}}{5!}\left(x - \frac{\pi}{4}\right)^5 + \dots$$

$$\sum_{n=0}^{\infty} \frac{\frac{\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right)^n (-1)^{\frac{n(n-1)}{2}}}{n!}$$