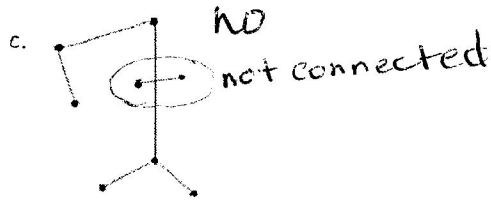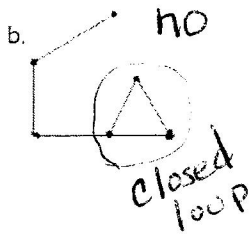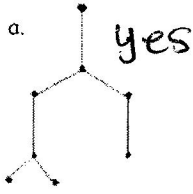# NOTES--Graph Theory Trees
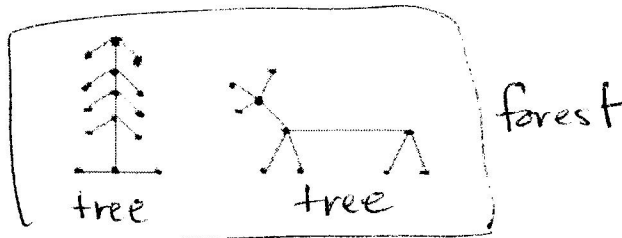
Trees

A **tree** is a simple, _Connected_ , _undirected_ graph with no closed loops.

Example 1 Is each graph a tree?
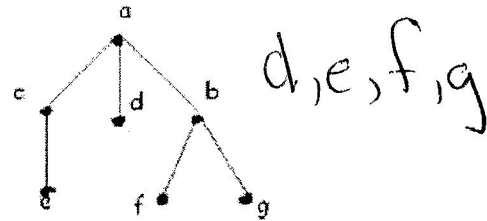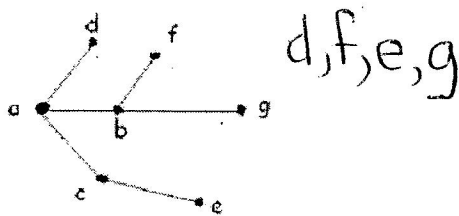
a. yes

b. no — closed loop

c. no — not connected

A _forest_ is a graph that consists of a set of trees.

tree   tree   forest

A _leaf_ is a vertex of degree "1".

A _root_ is a chosen vertex from which edges are directed.

Example 2 Let a be the root. List the leaves of a.

d, f, e, g

d, e, f, g

parent -- the next higher-level vertex

child(ren) -- the next lower-level vertex/vertices

_ancestors_ -- all vertices on the path back to the root

siblings -- vertices with the same parent

descendants -- vertices on the path below the given vertex

subtree -- includes the vertex and all of its descendants

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Example 3 Find:

a. the parent of c b

b. the children of g h, i, j

c. the siblings of h i, j

d. the ancestors of e a, b, c

e. the descendants of b c, d, e

f. the leaves d, e, f, i, k, l, m

g. the subtree rooted at g
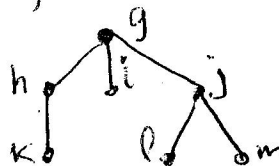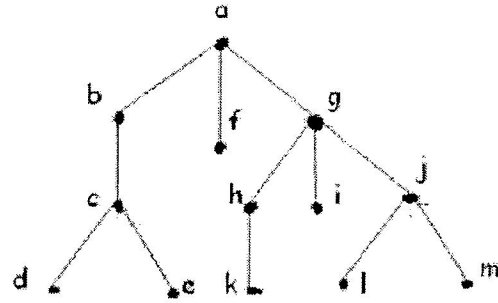
<u>internal vertices</u> -- vertices which have children

<u>terminal/external vertices</u> -- vertices which do not have children
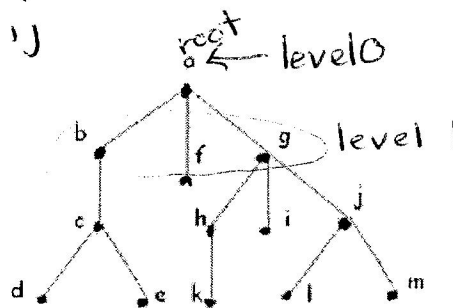
<u>level</u> -- the length of the path from the root to that vertex

<u>height</u> -- the highest level number

Example 4 Find:

internal vertices = a, b, c, g, h, j

level	vertices
0	a
1	b, f, g
2	c, h, i, j
3	d, e, k, l, m

height of the tree = 3

Binary Search Trees

In a binary tree search, you look for a ___key___ in a tree.
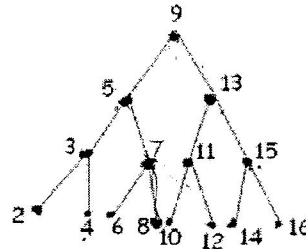
<u>Guidelines</u>:

1) Each vertex is labeled with a key.
2) All children are either "right" or "left"
3) Each key is greater than all the keys in its left subtree and is less than all the keys in its right subtree.
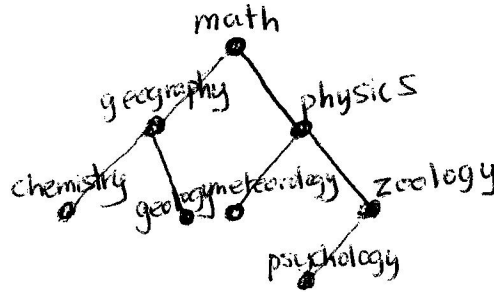
Example 5

a. search for 8 $\quad 9 \xrightarrow{\text{left}} 5 \xrightarrow{\text{right}} 7 \xrightarrow{\text{right}} 8$

b. search for 14 $\quad 9 \xrightarrow{R} 13 \xrightarrow{R} 15 \xrightarrow{L} 14$

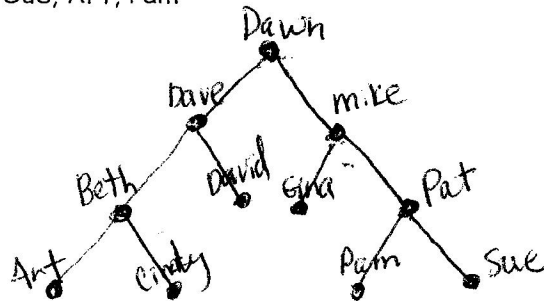c. search for 4 $\quad 9 \xrightarrow{L} 5 \xrightarrow{L} 3 \xrightarrow{R} 4$
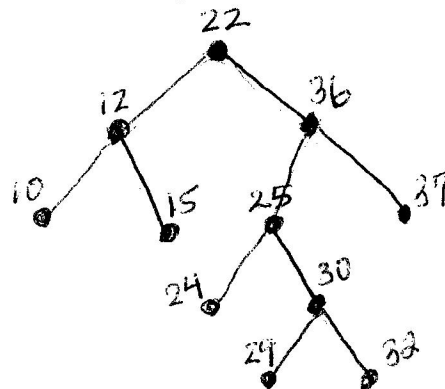

Example 6 Draw a tree for the following words in alphabetical order: math, physics, geography, zoology, meteorology, geology, psychology, chemistry.

root

Example 7 Draw a tree for the following names in alphabetical order: Dawn, Dave, Mike, David, Gina, Pat, Beth, Cindy, Sue, Art, Pam

root

Example 8 Draw a tree for the following class sizes in numerical order: 22, 36, 12, 15, 25, 36, 30, 29, 10, 37, 24, 32

repeat

root

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