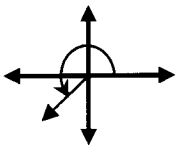
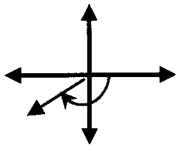
AP Calculus AB

Trigonometry Review



POSITIVE angles are generated by **COUNTER CLOCKWISE** rotations



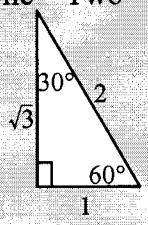
NEGATIVE angles are generated by **CLOCKWISE** rotations

We have several mnemonic devices to help us with trigonometry:

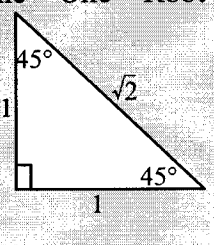
SohCahToa

$\sin\theta = \frac{\text{opp}}{\text{hyp}}$ $\cos\theta = \frac{\text{adj}}{\text{hyp}}$ $\tan\theta = \frac{\text{opp}}{\text{adj}}$

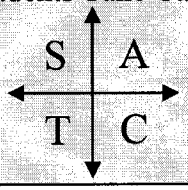
"One – Two – Root 3"



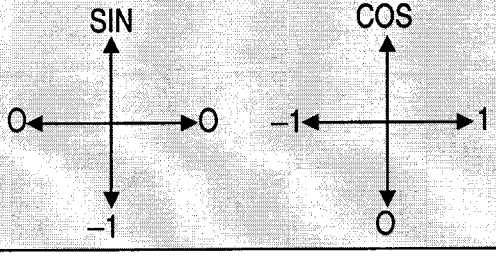
"One – One – Root 2"



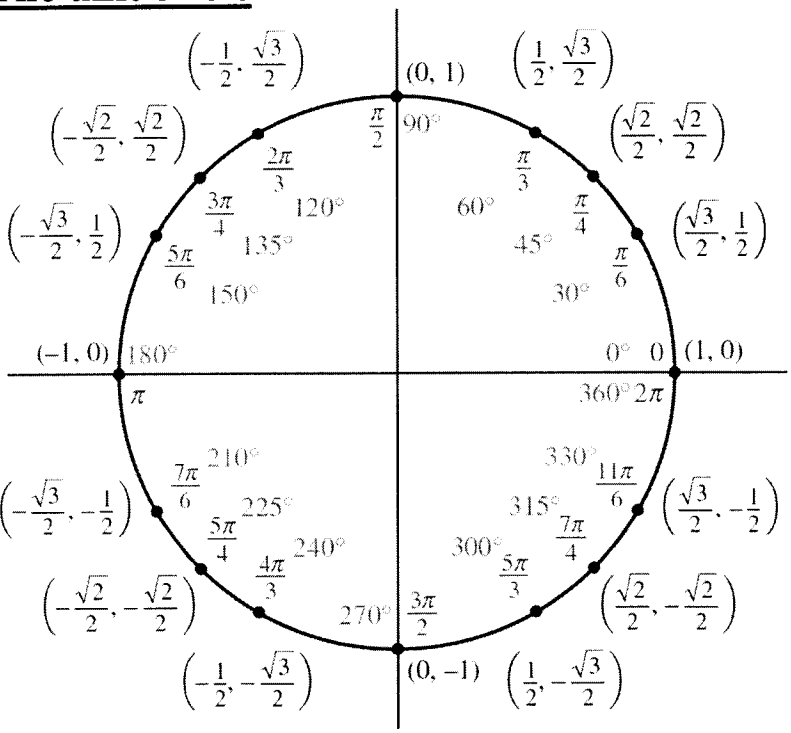
"All Students Take Calculus"



Quadrantal Angles: Remember that: $\tan\theta = \frac{\sin\theta}{\cos\theta}$



The unit circle:



$\sin t = y$ $\quad \csc t = \frac{1}{y} \ (y \neq 0)$

$\cos t = x$ $\quad \sec t = \frac{1}{x} \ (x \neq 0)$

$\tan t = \frac{y}{x} \ (x \neq 0)$ $\quad \cot t = \frac{x}{y} \ (y \neq 0)$

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Reciprocal Identities

$$\sin\theta = \frac{1}{\csc\theta} \qquad \csc\theta = \frac{1}{\sin\theta}$$

$$\cos\theta = \frac{1}{\sec\theta} \qquad \sec\theta = \frac{1}{\cos\theta}$$

$$\tan\theta = \frac{1}{\cot\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

Quotient Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

Pythagorean Identities

$$\cos^2\theta + \sin^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$\cot^2\theta + 1 = \csc^2\theta$$

Cofunction Identities

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta \qquad \cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta \qquad \tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta$$

$$\csc\left(\frac{\pi}{2} - \theta\right) = \sec\theta \qquad \sec\left(\frac{\pi}{2} - \theta\right) = \csc\theta \qquad \cot\left(\frac{\pi}{2} - \theta\right) = \tan\theta$$

Even-Odd Identities

$$\sin(-x) = -\sin x \qquad \csc(-x) = -\csc x$$

$$\cos(-x) = \cos x \qquad \sec(-x) = \sec x$$

$$\tan(-x) = -\tan x \qquad \cot(-x) = -\cot x$$

Sum & Difference Identities

$$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$$

Note the sign switch

$$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$$

Note that the sign does *not* switch

$$\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$$

Double Angle Identities

$$\sin 2u = 2 \sin u \cos u$$

$$\cos 2u = \begin{cases} \cos^2 u - \sin^2 u \\ 2\cos^2 u - 1 \\ 1 - 2\sin^2 u \end{cases}$$

$$\tan 2u = \frac{2\tan u}{1 - \tan^2 u}$$

Power Reducing Identities

$$\sin^2 u = \frac{1 - \cos 2u}{2}$$

$$\cos^2 u = \frac{1 + \cos 2u}{2}$$

$$\tan^2 u = \frac{1 - \cos 2u}{1 + \cos 2u}$$

Half Angle Identities

$$\sin\frac{u}{2} = \pm\sqrt{\frac{1 - \cos u}{2}}$$

$$\cos\frac{u}{2} = \pm\sqrt{\frac{1 + \cos u}{2}}$$

(whether the answer is positive or negative depends on what quadrant the input angle is in)

$$\tan\frac{u}{2} = \begin{cases} \pm\sqrt{\dfrac{1 - \cos u}{1 + \cos u}} \\ \dfrac{1 - \cos u}{\sin u} \\ \dfrac{\sin u}{1 + \cos u} \end{cases}$$

Inverse Trig Functions

$\sin^{-1}(y) = \theta$ or $\arcsin(y) = \theta$

Domain: $[-1, 1]$

Range: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

(quadrants I & IV)

$\cos^{-1}(y) = \theta$ or $\arccos(y) = \theta$

Domain: $[-1, 1]$

Range: $[0, \pi]$

(quadrants I & II)

$\tan^{-1}(y) = \theta$ or $\arctan(y) = \theta$

Domain: $(-\infty, \infty)$

Range: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(quadrants I & IV)