

NOTES--Graphs of Sine, Cosine and Tangent

VOCABULARY

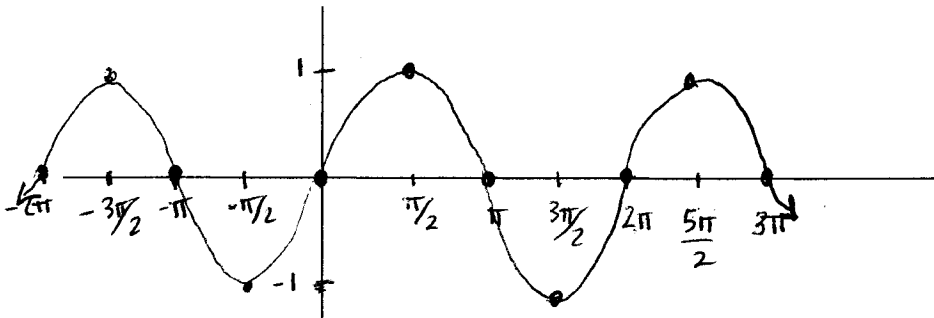
Periodic A function whose graph has a repeating pattern that continues indefinitely

Cycle The shortest repeating portion of a periodic function

Period The horizontal length of each cycle of a periodic function

Amplitude The amplitude of the graph of a sine or cosine function is $\frac{1}{2}(M - m)$ where M is the maximum value of the function and m is the minimum value of the function.

$$y = \sin \theta$$



domain: $(-\infty, \infty)$

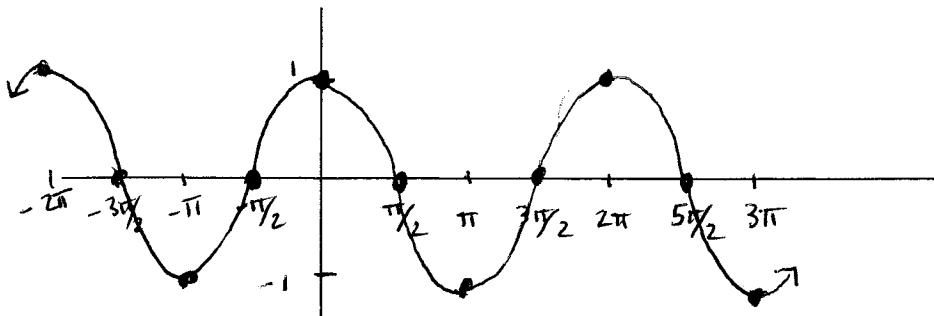
amplitude: 1 $\frac{1}{2}(1 - (-1))$

range: $[-1, 1]$

period: 2π

θ	$\sin \theta$
0	0
$\pi/2$	1
π	0
$3\pi/2$	-1
2π	0

$$y = \cos \theta$$



domain: $(-\infty, \infty)$

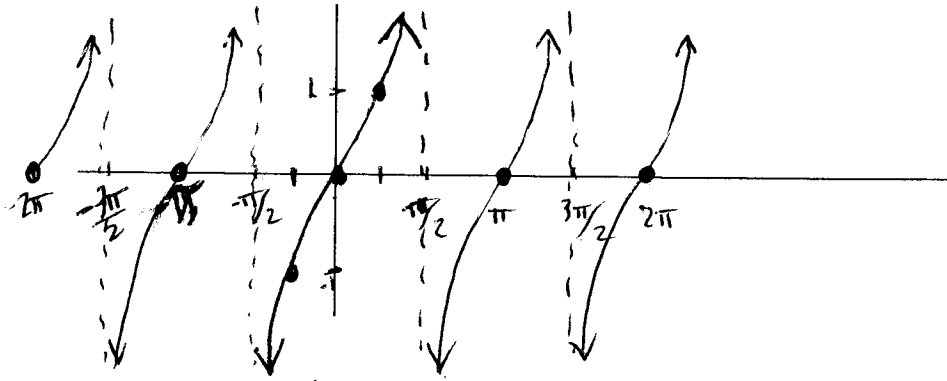
amplitude: 1

range: $[-1, 1]$

period: 2π

θ	$\cos \theta$
0	1
$\pi/2$	0
π	-1
$3\pi/2$	0
2π	1

$$y = \tan \theta$$



domain: $x \neq k \cdot \pi/2$
 k is an odd integer
 range: $(-\infty, \infty)$
 amplitude: none
 period: π

θ	$\tan \theta$
0	$\frac{0}{1} = 0$
$\pi/2$	$\frac{1}{0}$ undefined
π	$\frac{0}{-1} = 0$
$3\pi/2$	$\frac{-1}{0}$ undef.
2pi	$\frac{0}{1} = 0$
$\pi/4$	$\frac{\sqrt{2}}{2} = 1$
$3\pi/4$	$\frac{\sqrt{2}}{2}$
$5\pi/4$	$-\frac{\sqrt{2}}{2} = -1$
$7\pi/4$	$-\frac{\sqrt{2}}{2}$

The form $y = A \sin(Bx + C) + D$ is the general form of the sine function. From this general form of the *sine* function, the amplitude, horizontal, phase, and vertical shifts from the basic trigonometric forms can be determined.

- A: modifies the amplitude in the y direction above and below the center line
- B: influences the period and phase shift of the graph
- C: influences the phase shift of the graph
- D: shifts the center line of the graph on the y-axis

$$\text{Amplitude} = |a|$$

$$\text{Period} = \frac{2\pi}{b}$$

$$\text{Phase Shift} = -\frac{c}{b}$$

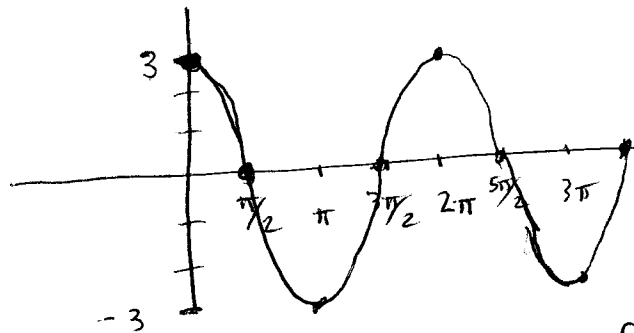
$$\text{Vertical Shift} = d$$

Example 1

** y's by 3*
 Graph $y = 3 \cos x$. Name the domain, range, amplitude, and period. Describe the transformations applied to the parent graph.

D: $(-\infty, \infty)$
 R: $[-3, 3]$
 amp = $|3| = 3$
 per = 2π

0	3
$\pi/2$	0
π	-3
$3\pi/2$	0
2π	3



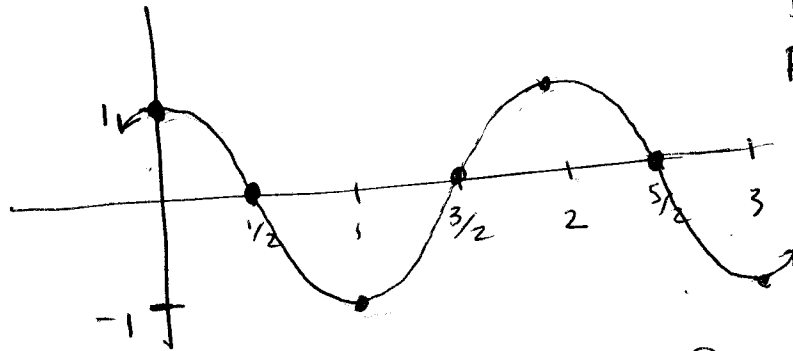
vertical stretch by a factor of 3

$\div x$'s by $\pi \Rightarrow *$ by $\frac{1}{\pi}$

Example 2

Graph $y = \cos x$. Name the domain, range, amplitude, and period. Describe the transformations applied to the parent graph.

0	1
$\frac{1}{2}$	0
1	-1
$\frac{3}{2}$	0
2	1



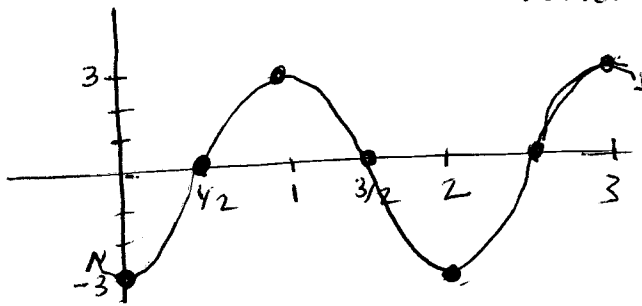
D: $(-\infty, \infty)$
 R: $[-1, 1]$
 amp = 1
 per = 2
 $\frac{2\pi}{\pi}$

*y's by -3 horizontal shrink by a factor of $\frac{1}{\pi}$
 $\div x$'s by π

Example 3

Graph $y = -3 \cos x$. Name the domain, range, amplitude, and period. Describe the transformations applied to the parent graph.

0	-3
$\frac{1}{2}$	0
1	3
$\frac{3}{2}$	0
2	-3



horiz. shrink by a factor of $\frac{1}{\pi}$
 vert. stretch by a factor of 3
 reflect over x-axis

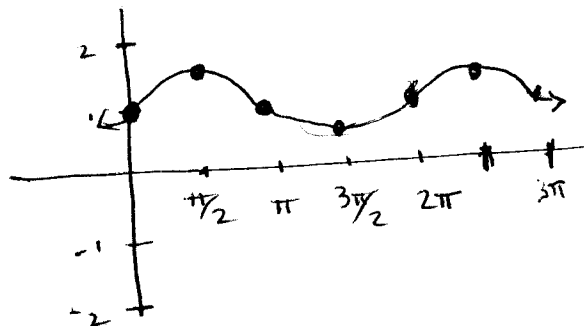
D: $(-\infty, \infty)$
 R: $[-3, 3]$
 amp = $|-3| = 3$
 per = $\frac{2\pi}{\pi} = 2$

*y's by $\frac{1}{2}$
 add 1 to y's

Example 4

Graph $y = \frac{1}{2} \sin(x) + 1$. Name the domain, range, amplitude, and period. Describe the transformations applied to the parent graph.

0	$0 + 1 = 1$
$\frac{\pi}{2}$	$\frac{1}{2} + 1 = \frac{3}{2}$
π	$0 + 1 = 1$
$\frac{3\pi}{2}$	$-\frac{1}{2} + 1 = \frac{1}{2}$
2π	$0 + 1 = 1$



vertical shrink by a factor of $\frac{1}{2}$
 vert. shift up 1

D: $(-\infty, \infty)$
 R: $[\frac{1}{2}, \frac{3}{2}]$
 amp = $|\frac{1}{2}| = \frac{1}{2}$
 per = $\frac{2\pi}{1} = 2\pi$

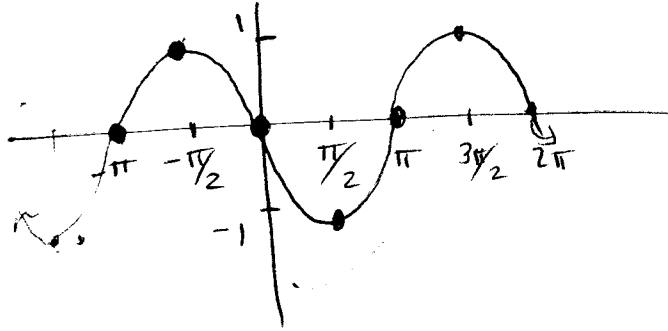
Example 5

Graph $y = \sin(x + \pi)$. Name the domain, range, amplitude, and period. Describe the transformations applied to the parent graph.

Subtract π from x 's shift to left π

phase shift = $-\frac{c}{b} = -\frac{\pi}{1} = -\pi$

$-\pi$	0
$-\pi/2$	1
0	0
$\pi/2$	-1
π	0



D: $(-\infty, \infty)$
 R: $[-1, 1]$
 amp = 1
 per = 2π

Example 6

Graph $y = \frac{1}{2} \sin(x + \pi) + 2$. Name the domain, range, amplitude, and period. Describe the transformations applied to the parent graph.

Example 7

Graph $y = -\tan\left(x - \frac{\pi}{2}\right)$. Name the domain, range, and period. Describe the transformations applied to the parent graph.