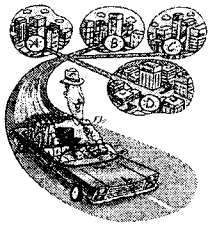
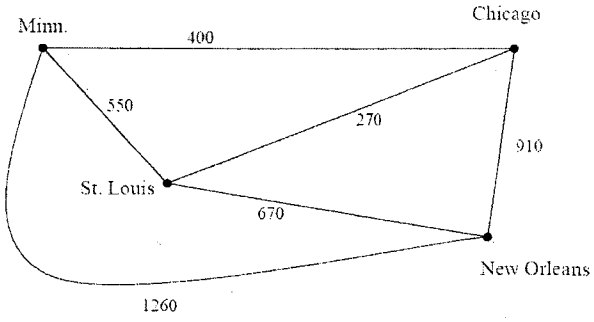
Notes Graph Theory Applications--Traveling Salesperson Problems



Traveling Salesperson Problems involve finding a Hamilton circuit of minimum value. This value could be time, distance, or cost.

In a weighted graph each edge is assigned a number which represents distance, cost, time, etc.

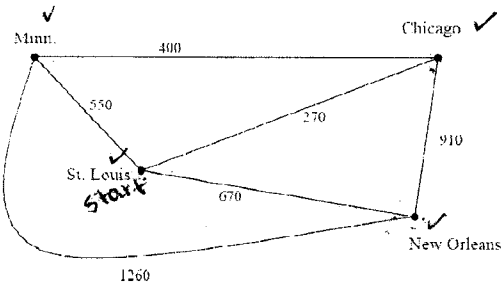


We will study two methods of solving TSP problems:

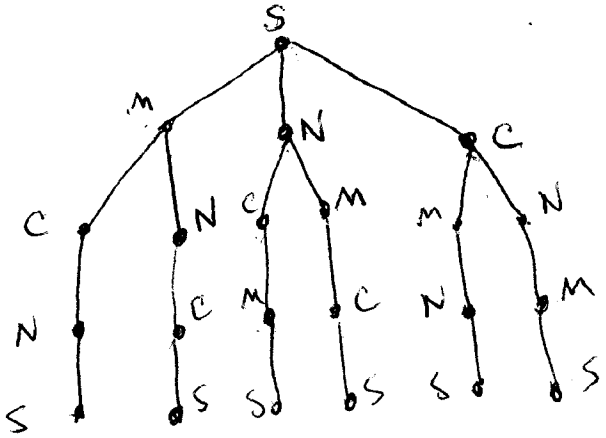
The Brute Force method produces the optimal solution. You list all of the possibilities and choose the one with the lowest value.

The Nearest-Neighbor method produces a close to optimal solution. You proceed through the lowest values along the way to produce a circuit. It is a quick way of finding a solution.

Example 1 A TSP lives in St. Louis and once a week travels to Minneapolis, Chicago, and New Orleans and then returns home. The graph represents the trips available to him, and the cost of the flights are shown on the graph. Find the least expensive route using the Brute Force method. Then apply the Nearest Neighbor method.



Brute Force



Nearest-Neighbor

SCMNS = 2600

270
400
1260
670

SMCNS = 2530

SMNCS = 2990

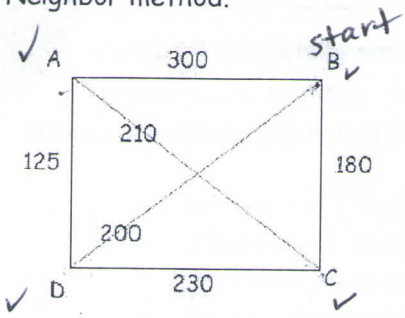
SNCMS = 2530

SNMCS = 2600

SCMNS = 2600

SCNMS = 2990

Example 2 The graph below represents distances between cities. If the salesman must begin and end in city B, find the shortest route using the Brute Force method. Then apply the Nearest Neighbor method.



B.F.

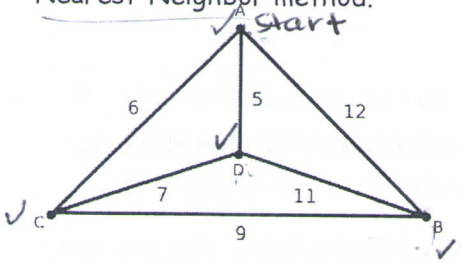
BACDB = 940
BADCB = 835
BDACB = 715
BDCAB = 940
BCADB = 715
BCDAB = 835

N.N

BCADB = 715

180
210
125
200

Example 3 Find the minimum Hamilton circuit (assume the starting point is A) using the Nearest-Neighbor method.



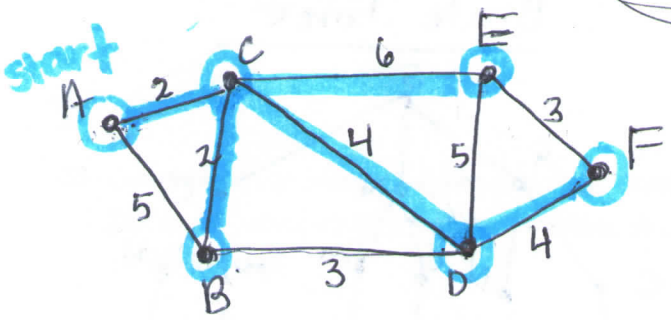
ADCBA

5
7
9
12

33

The Shortest path algorithm finds the shortest paths between a starting vertex and all of the other vertices of a graph. One algorithm process was developed by Edsger Wybe Dijkstra, a computer science dude who died in 2002.

Example 4 Find the shortest path from A to F. ACDF = 10



AC (2)
AB 5

AB 5
ACE 8
ACD 6
ACB (4)

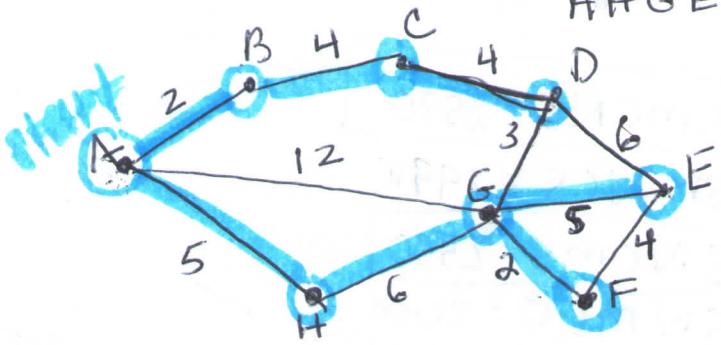
ACE 8
ACD (6)
ACBD 7

ACE (8)
ACDE 11
ACDF 10

ACEF 11
ACDF (10)

Example 5 Find the shortest path from A to E.

AHGE 16



AB (2)
AG 12
AH 5

AH (5)
AG 12
ABC 6

ABC (6)
AG 12
AHG 11

ABCD (10)
AG 12
AHG 11

ABCDE 16
ABCDG 13
AG 12
AHG (11)

ABCDE 16
AHGE 16
AHGF (13)

ABCDE 16 AHGE
AHGFE 17