NOTES--Graph Theory Types of Graphs

Types of Graphs

A Simple graph has no loops, multiple edges, or arrows.

A multigraph has multiple edges between the same vertices but no loops.

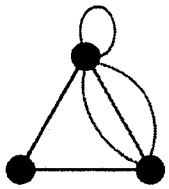
A pseudograph has multiple edges between the same vertices and loops.

A directed graph (digraph) has edges that are directed.

A directed multigraph is a directed graph with multiple edges and/or loops.

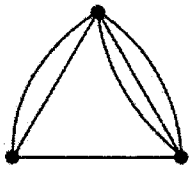
Example 1 Identify the type of graph:

A.



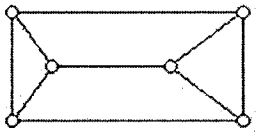
pseudograph

B.



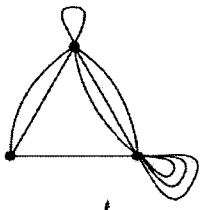
multigraph

C.



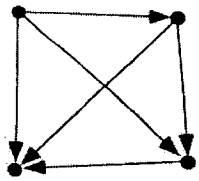
simple

D.



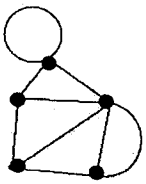
pseudograph

E.



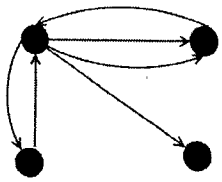
directed graph

F.



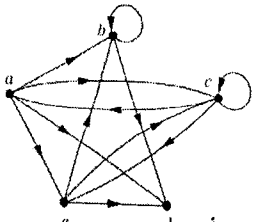
pseudograph

G.



directed multigraph

H.



directed multigraph

Directed Graphs

The in-degree for a directed graph is the number of edges with that vertex as their terminal vertex (how many edges come "into" that vertex).

The out-degree for a directed graph is the number of edges with that vertex as their initial vertex (how many edges go "out" of that vertex).

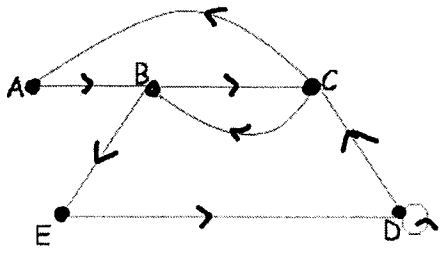
degree of vertex = sum of the in & out degrees

Things to know:

--A loop contributes to both the in-degree and out-degree of a vertex.

--The sum of the in-degree and out-degree of a vertex = the degree of the vertex.

Example 2



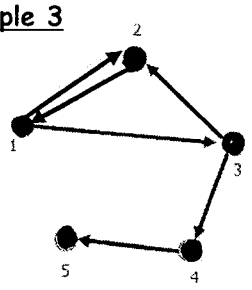
type of graph: directed multigraph

vertices = 5

edges = 8

VERTEX	IN-DEGREE	OUT-DEGREE	degree
A	1	1	2
B	2	2	4
C	2	2	4
D	2	2	4
E	1	1	2

Example 3



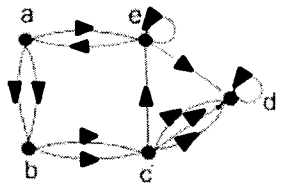
type of graph: directed multigraph

vertices = 5

edges = 6

vertex	in-degree	out-degree	degree
1	1	2	3
2	2	1	3
3	1	2	3
4	1	1	2
5	1	0	1

Example 4



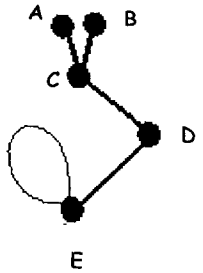
type of graph: directed multigraph

vertices = 5

edges = 13

vertex	in-degree	out-degree	degree
a	1	3	4
b	2	2	4
c	2	4	6
d	5	1	6
e	3	3	6

Example 5



type of graph: pseudograph

vertices = 5

edges = 5

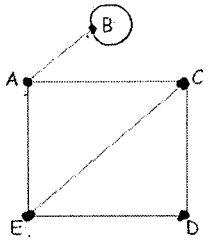
vertex	degree
A	1
B	1
C	3
D	2
E	3

Representations of Graphs

The endpoints of an edge are called adjacent vertices.

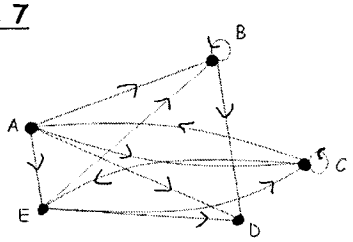
An adjacency list is a list of vertices specifying which ones are adjacent to one another.

Example 6



vertices	adjacent vertices
A	B, C, E
B	A, B
C	A, D, E
D	C, E
E	A, C, D

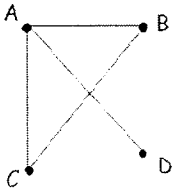
Example 7



vertices	adjacent vertices
A	B, C, D, E
B	B, D
C	A, C, E
D	none
E	B, C, D

An adjacency matrix is a table that encodes the structure of the graph.

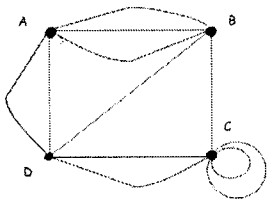
Example 8



4 vertices
4 × 4 matrix

	A	B	C	D
A	0	1	1	1
B	1	0	1	0
C	1	1	0	0
D	1	0	0	0

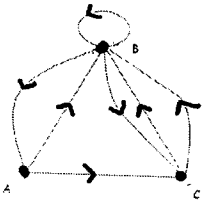
Example 9



4 vertices
4 × 4 matrix

	A	B	C	D
A	0	3	0	2
B	3	0	1	1
C	0	1	2	2
D	2	1	2	0

Example 10



3 vertices
3 × 3 matrix

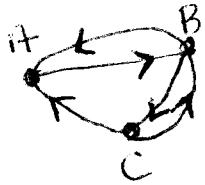
	A	B	C
A	0	1	1
B	1	1	1
C	0	1	0

Example 11

Given:

	A	B	C
A	0	1	0
B	1	0	1
C	1	1	0

Draw a directed graph.



Example 12

Given:

	A	B	C	D
A	0	3	0	2
B	3	0	1	0
C	0	1	1	2
D	2	0	2	0

Draw an undirected graph.