Integration By Substitution

review: $\int x^{12}\,dx = \frac{1}{13}x^{13} + C$

new: $\int (x^2+1)^{12} \cdot 2x\,dx$ yucky!

STEPS

① choose "u". – its deriv. must be in the problem
– try an inside function

② calculate "du"

③ substitute in u & du (all x's and dx must be replaced)

④ find anti-deriv.

⑤ substitute to get "x" stuff again

Evaluate

① $\int (x^2+1)^{12} \cdot 2x\,dx$

$u = x^2+1$

$\frac{du}{dx} = 2x$

$du = 2x\,dx$

$\int u^{12}\,du = \frac{1}{13}u^{13} + C$

$\boxed{\frac{1}{13}(x^2+1)^{13} + C}$

(2) $\int t^3 \sqrt{t^4+5}\, dt$

$u = t^4+5$

$\frac{du}{dt} = 4t^3$

$du = 4t^3 dt$

$\frac{1}{4} du = t^3 dt$

$\frac{1}{4}\int \sqrt{u}\, du$

$\frac{1}{4} \cdot \frac{2}{3} u^{3/2} + C$

$\boxed{\frac{1}{6}(t^4+5)^{3/2} + C}$

(3) $\int \frac{x}{(1-x^2)^4} dx$

$u = 1-x^2$

$\frac{du}{dx} = -2x$

$-\frac{1}{2} du = x\,dx$

$-\frac{1}{2}\int \frac{du}{u^4} = -\frac{1}{2}\int u^{-4} du$

$\frac{1}{6} u^{-3} + C$

$\frac{1}{6}(1-x^2)^{-3} + C$

$\boxed{\frac{1}{6(1-x^2)^3} + C}$

(4) $\int x \sin x^2\, dx$

$u = x^2$

$\frac{du}{dx} = 2x$

$\frac{1}{2} du = x\,dx$

$\frac{1}{2}\int \sin u\, du$

$-\frac{1}{2}\cos u + C$

$\boxed{-\frac{1}{2}\cos(x^2) + C}$

(5) $\int \tan^7 x \sec^2 x \, dx$

$u = \tan x$

$\frac{du}{dx} = \sec^2 x$

$du = \sec^2 x \, dx$

$\int u^7 \, du$

$\frac{1}{8} u^8 + C$

$\frac{1}{8} (\tan x)^8 + c$

$\boxed{\frac{1}{8} \tan^8 x + C}$

(6) $\int \frac{2x+1}{\sqrt{x+4}} dx$

$u = x+4 \rightarrow x = u-4$

$\frac{du}{dx} = 1$

$du = dx$

$\int \frac{2(u-4)+1}{\sqrt{u}} du$

$\int \frac{2u - 8 + 1}{\sqrt{u}} du$

$\int (2u^{1/2} - 7u^{-1/2}) du$

$\frac{4}{3} u^{3/2} - 14u^{1/2} + c$

$\boxed{\frac{4}{3}(x+4)^{3/2} - 14(x+4)^{1/2} + C}$

7) $\int (x+7)\sqrt[3]{3-2x}\,dx$

$u = 3-2x \rightarrow u-3 = -2x$

$\frac{du}{dx} = -2 \qquad \frac{u-3}{-2} = x$

$-\frac{1}{2}du = dx$

$$-\frac{1}{2}\int\left(\frac{u-3}{-2}+7\right)\sqrt[3]{u}\,du$$

$$-\frac{1}{2}\int\left(-\frac{1}{2}u+\underbrace{\frac{3}{2}+7}_{\frac{17}{2}}\right)\sqrt[3]{u}\,du$$

$$-\frac{1}{2}\int\left(-\frac{1}{2}u^{4/3}+\frac{17}{2}u^{1/3}\right)du$$

$$-\frac{1}{2}\left(-\frac{3}{14}u^{7/3}+\frac{51}{8}u^{4/3}+C\right)$$

$$\boxed{\frac{3}{28}(3-2x)^{7/3}-\frac{51}{16}(3-2x)^{4/3}+C}$$

8) $\int_{-2}^{4} x^2(x^3+8)^2\,dx$

$x=-2,\ u=(-2)^3+8=0$

$x=4,\ u=(4)^3+8=72$

$u = x^3+8 \qquad \frac{1}{3}\int u^2\,du$

$\frac{du}{dx} = 3x^2 \qquad \frac{1}{9}u^3+C$

$\frac{1}{3}du = x^2dx$

$$\frac{1}{9}(x^3+8)^3+C\Big|_{-2}^{4}$$

$$\frac{72^3}{9}-(0) = \boxed{41472}$$

$$\frac{1}{3}\int_0^{72}u^2\,du$$

$$\frac{1}{9}u^3+C\Big|_0^{72}$$

$$\frac{1}{9}(72)^3-0$$

41472