U- Substitution (Day 2)

EX1 Evaluate:

(A) $\int_0^{\pi/4} \cos(\pi - x)\,dx$

$u = \pi - x$

$\frac{du}{dx} = -1 \qquad -du = dx$

$-\int \cos u \, du$

$-\sin u + C$

$-\sin(\pi - x) + C \Big|_0^{\pi/4}$

$-\sin(3\pi/4) + c - [-\sin(\pi) + C]$

$\boxed{-\frac{\sqrt{2}}{2}}$

Another way...

if $x = 0$, $u = \pi - x = \pi - 0 = \pi$

if $x = \pi/4$, $u = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$

So we could use $-\sin u + c$ with these values

$-\sin u + c \Big|_{\pi}^{3\pi/4}$

$-\sin\frac{3\pi}{4} + c - [-\sin\pi + c]$

$= \boxed{-\frac{\sqrt{2}}{2}}$

(B) $\int \sin^5(2x)\cos(2x)\,dx$

$u = \sin(2x)$

$\frac{du}{dx} = 2\cos(2x) \qquad \frac{1}{2}du = \cos(2x)dx$

$\frac{1}{2}\int u^5 \, du = \frac{1}{12}u^6 + C$

$= \frac{1}{12}(\sin(2x))^6 + C \Big|_0^{\pi/8}$

$= \frac{1}{12}\left(\sin\left(\frac{\pi}{4}\right)\right)^6 + c - \left[\frac{1}{12}(\sin(0))^6 + c\right]$

$= \frac{1}{12}\left(\frac{\sqrt{2}}{2}\right)^6 = \frac{1}{12} \cdot \frac{8}{64} = \boxed{\frac{1}{96}}$

(c) $\int_0^2 x(x^2+1)^3\,dx$

$u = x^2+1$

$\frac{du}{dx} = 2x \qquad \frac{1}{2}du = x\,dx$

$\frac{1}{2}\int u^3\,du = \frac{1}{2}\cdot\frac{1}{4}u^4 + C$

$= \frac{1}{8}(x^2+1)^4 + C \Big|_0^2$

$= \frac{1}{8}(5)^4 + C - \left[\frac{1}{8}(1)^4 + C\right]$

$= \frac{625}{8} - \frac{1}{8} = \frac{624}{8} = \boxed{78}$

<u>EX 2</u> Find $f(x)$ if $f'(x) = \sec^2(2x)$ and f passes through $(\pi/2, 2)$.

$f(x) = \int \sec^2(2x)\,dx$

$u = 2x$

$\frac{du}{dx} = 2 \qquad \frac{1}{2}du = dx$

$\frac{1}{2}\int \sec^2 u\,du$

$\frac{1}{2}\tan u + C$

$f(x) = \frac{1}{2}\tan(2x) + C$

$2 = \frac{1}{2}\tan(2\cdot\pi/2) + C$

$2 = \frac{1}{2}(0) + C$

$2 = C$

$\boxed{f(x) = \frac{1}{2}\tan(2x) + 2}$