1990 BC3

Let $f(x) = 12 - x^2$ for $x \geq 0$ and $f(x) \geq 0$.

(a) The line tangent to the graph of f at the point $(k, f(k))$ intercepts the x-axis at $x = 4$. What is the value of k ?

(b) An isosceles triangle whose base is the interval from $(0,0)$ to $(c,0)$ has its vertex on the graph of f. For what value of c does the triangle have maximum area? Justify your answer.

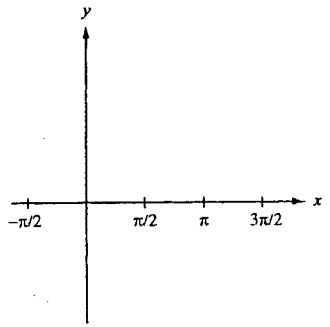
1975 AB4/BC1

Given the function defined by $y = x + \sin x$ for all x such that $-\frac{\pi}{2} \leq x \leq \frac{3\pi}{2}$.

(a) Find the coordinates of all maximum and minimum points on the given interval. Justify your answers.

(b) Find the coordinates of all points of inflection on the given interval. Justify your answers.

(c) On the axes provided, sketch the graph of the function.



1978 AB5/BC1

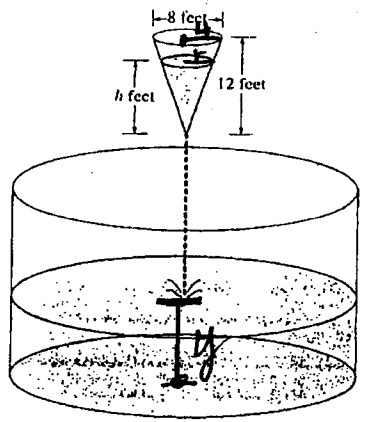
Given the curve $x^2 - xy + y^2 = 9$.

(a) Write a general expression for the slope of the curve.

(b) Find the coordinates of the points on the curve where the tangents are vertical.

(c) At the point $(0,3)$ find the rate of change in the slope of the curve with respect to x.

1995 AB5/BC3



As shown in the figure above, water is draining from a conical tank with height 12 feet and diameter 8 feet into a cylindrical tank that has a base with area 400π square feet. The depth h, in feet, of the water in the conical tank is changing at the rate of $(h-12)$ feet per minute. (The volume V of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.)

circle

$\frac{dh}{dt}$

(a) Write an expression for the volume of water in the conical tank as a function of h.

(b) At what rate is the volume of water in the conical tank changing when $h = 3$? Indicate units of measure.

(c) Let y be the depth, in feet, of the water in the cylindrical tank. At what rate is y changing when $h = 3$? Indicate units of measure.

a) $V = \frac{1}{3}\pi r^2 h$

$\frac{r}{h} = \frac{4}{12}$

$12r = 4h$

$r = \frac{4h}{12} = \frac{h}{3}$

$V = \frac{1}{3}\pi\left(\frac{h}{3}\right)^2 \cdot h = \frac{\pi h^3}{27}$ ft^3

b) Find $\frac{dV}{dt}$ water when $h = 3$

$V = \frac{\pi h^3}{27}$

$\frac{dV}{dt} = \frac{\pi}{27} \cdot 3h^2 \frac{dh}{dt}$

$\frac{dV}{dt} = \frac{\pi}{27} \cdot 3(3)^2 (h-12)$ ($h = 3$)

$\frac{dV}{dt} = -9\pi$ ft^3/min

c) Find $\frac{dy}{dt}$ when $h = 3$

$V = \pi r^2 h$

$V = 400\pi y$

$\frac{dV}{dt} = 400\pi \cdot \frac{dy}{dt}$

$9\pi = 400\pi \frac{dy}{dt}$

$\frac{dy}{dt} = \frac{9}{400}$ ft/min