1989 AB3
Solution

(a) $v(t) = \int 4\cos 2t\,dt$

$v(t) = 2\sin 2t + C$

$v(0) = 1 \Rightarrow C = 1$

$v(t) = 2\sin 2t + 1$

(b) $x(t) = \int 2\sin 2t + 1\;dt$

$x(t) = -\cos 2t + t + C$

$x(0) = 0 \Rightarrow C = 1$

$x(t) = -\cos 2t + t + 1$

(c) $2\sin 2t + 1 = 0$

$\sin 2t = -\frac{1}{2}$

$2t = \frac{7\pi}{6}, \frac{11\pi}{6}$

$t = \frac{7\pi}{12}, \frac{11\pi}{12}$

1994 AB 6

(a) $F(1) = \int_0^1 \sin\left(t^2\right) dt$

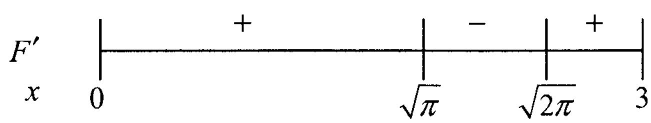
$$\approx \frac{(1-0)}{4} \cdot \frac{1}{2} \cdot \left[\sin 0^2 + 2\sin\left(\frac{1}{4}\right)^2 + 2\sin\left(\frac{1}{2}\right)^2 + 2\sin\left(\frac{3}{4}\right)^2 + \sin 1^2 \right]$$

$$\approx 0.316$$

(b) $F'(x) = \sin\left(x^2\right)$

$F'(x) = 0$ when $x^2 = 0, \pi, 2\pi, \ldots$

$x = 0, \sqrt{\pi}, \sqrt{2\pi}$



F is increasing on $\left[0, \sqrt{\pi}\right]$ and on $\left[\sqrt{2\pi}, 3\right]$

(c)

$$k = \frac{F(3) - F(1)}{2} = \frac{\int_1^3 \sin\left(t^2\right) dt}{2}$$

$$\int_1^3 \sin\left(t^2\right) dt = 2k$$

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Question 2

t (hours)	0	1	3	4	7	8	9
$L(t)$ (people)	120	156	176	126	150	80	0

Concert tickets went on sale at noon ($t = 0$) and were sold out within 9 hours. The number of people waiting in line to purchase tickets at time t is modeled by a twice-differentiable function L for $0 \le t \le 9$. Values of $L(t)$ at various times t are shown in the table above.

(a) Use the data in the table to estimate the rate at which the number of people waiting in line was changing at 5:30 P.M. ($t = 5.5$). Show the computations that lead to your answer. Indicate units of measure.

(b) Use a trapezoidal sum with three subintervals to estimate the average number of people waiting in line during the first 4 hours that tickets were on sale.

(c) For $0 \le t \le 9$, what is the fewest number of times at which $L'(t)$ must equal 0 ? Give a reason for your answer.

(d) The rate at which tickets were sold for $0 \le t \le 9$ is modeled by $r(t) = 550te^{-t/2}$ tickets per hour. Based on the model, how many tickets were sold by 3 P.M. ($t = 3$), to the nearest whole number?

(a) $L'(5.5) \approx \dfrac{L(7) - L(4)}{7 - 4} = \dfrac{150 - 126}{3} = 8$ people per hour

$2: \begin{cases} 1: \text{estimate} \\ 1: \text{units} \end{cases}$

(b) The average number of people waiting in line during the first 4 hours is approximately

$$\frac{1}{4}\left(\frac{L(0) + L(1)}{2}(1 - 0) + \frac{L(1) + L(3)}{2}(3 - 1) + \frac{L(3) + L(4)}{2}(4 - 3)\right)$$

$= 155.25$ people

$2: \begin{cases} 1: \text{trapezoidal sum} \\ 1: \text{answer} \end{cases}$

(c) L is differentiable on $[0, 9]$ so the Mean Value Theorem implies $L'(t) > 0$ for some t in $(1, 3)$ and some t in $(4, 7)$. Similarly, $L'(t) < 0$ for some t in $(3, 4)$ and some t in $(7, 8)$. Then, since L' is continuous on $[0, 9]$, the Intermediate Value Theorem implies that $L'(t) = 0$ for at least three values of t in $[0, 9]$.

$3: \begin{cases} 1: \text{considers change in sign of } L' \\ 1: \text{analysis} \\ 1: \text{conclusion} \end{cases}$

OR

The continuity of L on $[1, 4]$ implies that L attains a maximum value there. Since $L(3) > L(1)$ and $L(3) > L(4)$, this maximum occurs on $(1, 4)$. Similarly, L attains a minimum on $(3, 7)$ and a maximum on $(4, 8)$. L is differentiable, so $L'(t) = 0$ at each relative extreme point on $(0, 9)$. Therefore $L'(t) = 0$ for at least three values of t in $[0, 9]$.

$3: \begin{cases} 1: \text{considers relative extrema of } L \text{ on } (0, 9) \\ 1: \text{analysis} \\ 1: \text{conclusion} \end{cases}$

[Note: There is a function L that satisfies the given conditions with $L'(t) = 0$ for exactly three values of t.]

(d) $\int_0^3 r(t)\,dt = 972.784$

There were approximately 973 tickets sold by 3 P.M.

$2: \begin{cases} 1: \text{integrand} \\ 1: \text{limits and answer} \end{cases}$