

Review Unit 4

D 1. slope =  $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{4 \cos t}{-3 \sin t} \Big|_{t=13} = \frac{4 \cos 13}{-3 \sin 13} = -\frac{4}{3} \cot 13 = \frac{-4}{3 \tan 13}$

C 2. at rest means no horiz or vert movement

$$\frac{dy}{dt} = 6t^2 - 6t - 12 \quad \frac{dx}{dt} = 3t^2 - 6t$$

$$6(t^2 - t - 2) = 0 \quad 3t(t - 2) = 0$$

$$6(t-2)(t+1) = 0 \quad t = 0 \quad t = 2$$

$$t = 2, -1$$

$L = 2$

A 3. slope =  $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{3t^2}{2t-4} \Big|_{t=2} = \frac{12}{0}$  undefined  
 when  $y = 8, t = 13 \Rightarrow t = 2$   $x = -3$

C 4. speed =  $|v(t)| = \sqrt{(4 \cos(4t))^2 + (2t)^2} \Big|_{t=3}$   
 $= \sqrt{(4 \cos 12)^2 + (6)^2}$

B 5.  $4 \sin 2\theta = 0$   
 $\sin 2\theta = 0$   
 $2\theta = \sin^{-1}(0)$   
 $2\theta = 0, \pi, 2\pi, 3\pi, 4\pi$   
 $\theta = 0, \pi/2, \pi, 3\pi/2, 2\pi$

$$\frac{1}{2} \int_{3\pi/2}^{2\pi} (4 \sin 2\theta)^2 d\theta$$

$$8 \int_{3\pi/2}^{2\pi} 16 \sin^2(2\theta) d\theta$$

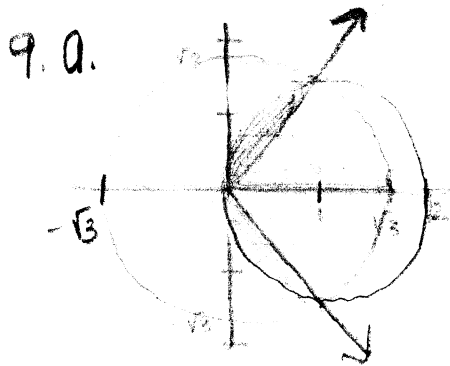
$$8 \int_{3\pi/2}^{2\pi} \sin^2(2\theta) d\theta$$

A 6.  $\int_0^{\pi} \sqrt{(1 + \cos \theta)^2 + (-\sin \theta)^2} d\theta$

D 7.  $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{4}{3t^2} = \frac{4}{3} t^{-2}$   $x = 2 - 11t^2 \Rightarrow t = 1$   
 $\frac{d^2y}{dx^2} = \frac{d(dy'/dt)}{dx/dt} = \frac{d(\frac{4}{3} t^{-2})/dt}{3t^2} = \frac{-\frac{8}{3} t^{-3}}{3t^2} = \frac{-8}{9t^5}$

$$A. 8. |v| = \left\langle \frac{1}{t^2+2t} \cdot 2t+2, 4t \right\rangle \Big|_{t=2}$$

$$= \left\langle \frac{1}{8} \cdot 6, 8 \right\rangle = \left\langle \frac{3}{4}, 8 \right\rangle$$



b.

$$2 \cos \theta = \frac{\sqrt{3}}{2}$$

$$\cos \theta = \frac{\sqrt{3}}{4}$$

$$\theta = \cos^{-1} \left( \frac{\sqrt{3}}{2} \right)$$

$$\theta = \pi/6, 11\pi/6$$

$$2 \left( \frac{1}{2} \int_0^{\pi/6} (\sqrt{3})^2 d\theta \right) + 2 \left( \frac{1}{2} \int_{11\pi/6}^{\pi/2} (2 \cos \theta)^2 d\theta \right)$$

$$1.571 + 1.228$$

$$\boxed{2.799}$$

10. a.  $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{3t^2 - 12}{6t^2 - 6t} = \frac{3(t^2 - 4)}{6t(t-1)} = \frac{t-4}{2(t-1)}$

b.  $\frac{dy}{dx} \Big|_{t=-1} = \frac{(-1)^2 - 4}{2((-1)^2 - (-1))} = \frac{-3}{4}$

if  $t = -1$ ,  $x = 2(-1)^3 - 3(-1)^2 = -2 - 3 = -5$

$y = (-1)^3 - 12(-1) = -1 + 12 = 11$

$$\boxed{y - 11 = -\frac{3}{4}(x + 5)}$$

vert:  $(0, 0)$   $(-1, -11)$

c.  $\frac{dy}{dx} = \frac{t^2 - 11}{2t(t-1)}$

horiz:  $(4, -16)$   $(-23, 16)$

undefined:  $t = 0, t = 1$

zero:  $t = \pm 3$

12. a. accel:  $\left\langle 12-6t, \frac{1}{1+(t-4)^4} \cdot 4(t-4)^3(1) \right\rangle \Big|_{t=2}$

$\langle 0, -32/17 \rangle = \langle 0, -1.882 \rangle$

speed:  $\sqrt{(12-31)^2 + (\ln(1+(t-4)^4))} \Big|_{t=2}$

$\sqrt{144 + (\ln 17)^2} = 12.330$

b.  $\int_0^2 \ln(1+(t-4)^4) dt = 8.671$   $\rightarrow$   $P(2) - P(0)$

start (0, y(0)=5)  $\rightarrow$   $\boxed{3.671}$

$$c. \frac{dy}{dx} = \frac{dy/dt}{dx/dt} \Big|_{t=2} = \frac{\ln 17}{12} = .236$$

point (3, 13.671)

$$y - 13.671 = \overset{.236}{\uparrow} \frac{\ln 17}{12} (x - 3)$$

d. rest  $\Rightarrow$  horiz & vert motion = 0

$$\frac{dx}{dt} = 12t - 3t^2 = 0$$

$$3t(4-t) = 0$$

$$t = 0, t = 4$$

$$\boxed{t = 4}$$

$$\frac{dy}{dt} = \ln(1 + (t-4)^4) = 0$$

$$1 + (t-4)^4 = 1$$

$$t = 4$$

new #11:

$$x = \frac{1}{t} \quad y = \frac{t}{1+t}$$

$$t = \frac{1}{x}$$

$$y = \frac{\frac{1}{x}}{1 + \frac{1}{x}} = \frac{1}{x+1}$$

v.a. at  $x = -1$

h.a. at  $y = 0$