ICM Exponential and Logarithmic Functions Test Review

Show your work for each problem.

1. Graph $f(x) = (3)^{x-2} - 1$

x	y
0	$-\frac{8}{9}$
1	$-\frac{2}{3}$
2	0
3	2
4	8

domain: $(-\infty, \infty)$

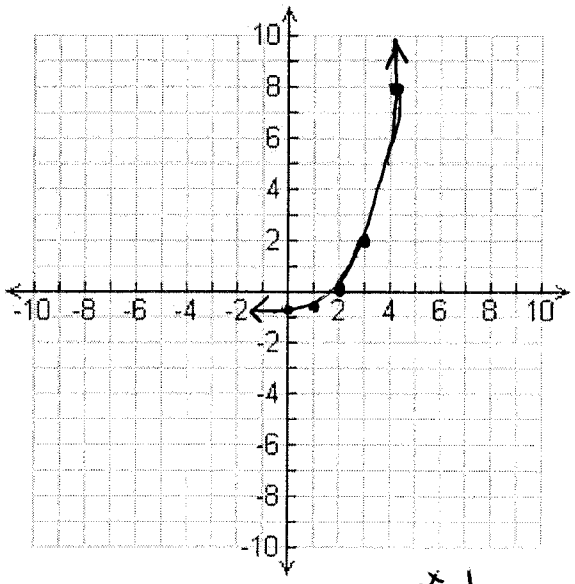
range: $(-1, \infty)$

horizontal asymptote: $y = -1$

describe the transformations on $y = 3^x$ to obtain the graph of f(x) right 2 units, down 1 unit

describe the end behavior

$\lim_{x \to -\infty} f(x) = -1$ $\lim_{x \to \infty} f(x) = \infty$



2. Graph $f(x) = \log_2(x-1)$ inverse: $x = \log_2(y-1)$

$2^x = y - 1$

$2^x + 1 = y$

x	y
-2	$\frac{5}{4}$
-1	$\frac{3}{2}$
0	2
1	3
2	5
3	9

$\frac{5}{4}$	-2
$\frac{3}{2}$	-1
2	0
3	1
5	2
9	3

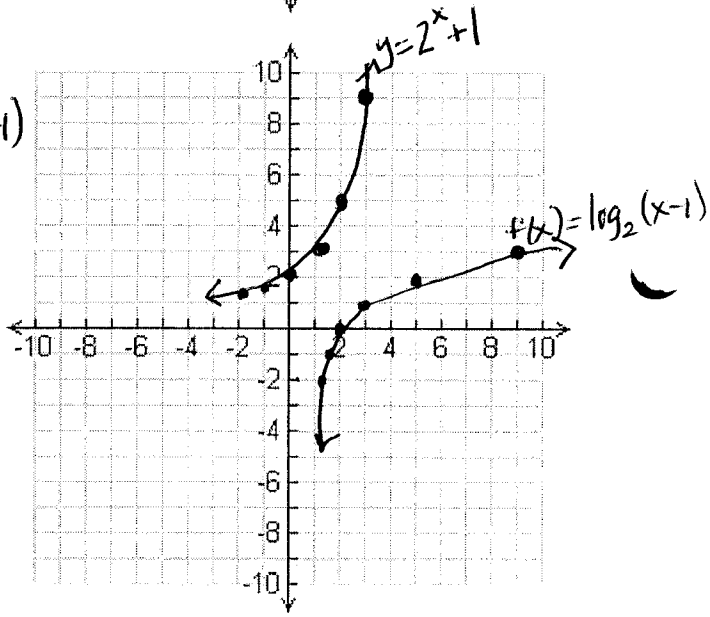
domain: $(1, \infty)$

range: $(-\infty, \infty)$

vertical asymptote: $x = 1$

describe the end behavior

$\lim_{x \to 1^+} f(x) = -\infty$ $\lim_{x \to \infty} f(x) = \infty$



3. Evaluate each expression without a calculator:

a. $\log_3\left(\frac{1}{81}\right) = -4$ b. $\ln e^{-4} = -4$

$3^x = \frac{1}{81} = 3^{-4}$ $e^x = e^{-4}$

$x = -4$ $x = -4$

4. Write as a single log: $3\log_4 x^2 + \frac{1}{2}\log_4 \sqrt{x}$

$\log_4 x^6 + \log_4 x^{1/4}$

$\log_4 (x^6 \cdot x^{1/4}) = \log_4 x^{25/4}$

5. Solve each equation algebraically:

a. $4^{x-1} = 2$

$(2^2)^{x-1} = 2$

$2^{2x-2} = 2^1$

$2x - 2 = 1$

$x = 3/2$

b. $\log_6(x+3) + \log_6(x+4) = 1$

$\log_6 (x^2 + 7x + 12) = 1$

$x^2 + 7x + 12 = 6^1$

$x^2 + 7x + 6 = 0$

$(x+6)(x+1) = 0$

$x = -6$ $x = -1$

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6. Solve with a calculator: $5^{x+2} = 7^{x-2}$ * change window if needed

$X = 21.133$

7. An amplifier's power output P (in watts) is related to its decibel voltage gain d by the formula $P = 25e^{0.1d}$. Find the power output for a decibel voltage gain of 4 decibels. For a power output of 50 watts, what is the decibel voltage gain?

$P = 25e^{0.1(4)} =$ 37.296 watts

$50 = 25e^{0.1d}$

$d = 6.931$

8. A child's grandparents wish to purchase a bond fund that matures in 18 years to be used for her college education. The bond fund pays 4% interest compounded semiannually. How much should they purchase so that the bond fund will be worth ~~$85,000 at maturity~~?

$85000 = P\left(1 + \frac{.04}{2}\right)^{18 \cdot 2}$

$P = \$41668.97$

9. A school has 1200 students. A rumor begins to spread, modeled by the logistic equation $R(t) = \frac{1200}{1 + 39e^{-0.9t}}$ where t = 0 is the day the rumor begins to spread & R(t) represents the number of students who have heard the rumor by the end of t days.

a. How many students initially heard the rumor? $R(0) = 30$ students

b. How many students have heard the rumor by the end of day 2? $R(2) = 161$ students

c. How long does it take for 1000 students to hear the rumor? 5.859 days

d. How many students can be predicted to eventually hear the rumor?

1200 students

10. The Richter scale is used to convert seismographic readings into numbers for measuring the magnitude of an earthquake. All earthquakes are compared to a zero-level earthquake whose seismographic reading measures 0.001 millimeter at a distance of 100 kilometers from the epicenter. An earthquake whose reading measures x millimeters has magnitude $M(x) = \log\left(\frac{x}{x_0}\right)$, where $x_0 = 10^{-3}$ is the reading of a zero-level earthquake that same distance from its epicenter. Find the magnitude of an earthquake whose seismographic reading is 10.0 millimeters at a distance of 100 kilometers from its epicenter.

$M(10.0) = \log\left(\frac{10.0}{10^{-3}}\right) =$ 4

11. The table below represents the population, y, of a herd of animals on an island over a period of x days.

a. Calculate a logistic model to represent the data. $y = \frac{99.884}{1 + 19.671e^{-.199x}}$

b. What is the maximum sustainable population for the island? 99.884 $\Rightarrow$ almost 100 animals

c. Based on the model, what is the population when the population is growing the fastest?

x	0	10	15	20	25	30
y	5	27	50	73	88	95

$\frac{1}{2}$ of carrying capacity $= 50$

$50 = \frac{99.884}{1 + 19.671e^{-.199x}}$

$x = 14.993$

12. Calculate an exponential regression equation for the data below.

$y = .800 * 1.400^x$

x	1	3	5	7	9
y	1.120	2.195	4.303	8.433	16.529

a. Using the model, find the y-value for an x-value of 11. $.800 * 1.400^{11} =$ 32.398

b. Using the model, find the x-value for a y-value of 38. $38 = .800 * 1.400^x$

$x = 11.474$