\#1 2004 #3

(a) $x(4) = x(2) + \int_2^4 \left(3 + \cos\left(t^2\right)\right) dt$

$= 1 + \int_2^4 \left(3 + \cos\left(t^2\right)\right) dt = 7.132$ or 7.133

$3 : \begin{cases} 1 : \int_2^4 \left(3 + \cos\left(t^2\right)\right) dt \\ 1 : \text{handles initial condition} \\ 1 : \text{answer} \end{cases}$

(b) $\left.\frac{dy}{dx}\right|_{t=2} = \left.\frac{\frac{dy}{dt}}{\frac{dx}{dt}}\right|_{t=2} = \frac{-7}{3 + \cos 4} = -2.983$

$y - 8 = -2.983(x - 1)$

$2 : \begin{cases} 1 : \text{finds } \left.\frac{dy}{dx}\right|_{t=2} \\ 1 : \text{equation} \end{cases}$

(c) The speed of the object at time $t = 2$ is $\sqrt{(x'(2))^2 + (y'(2))^2} = 7.382$ or 7.383.

1 : answer

(d) $x''(4) = 2.303$

$y'(t) = \frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt} = (2t + 1)\left(3 + \cos\left(t^2\right)\right)$

$y''(4) = 24.813$ or 24.814

The acceleration vector at $t = 4$ is $\langle 2.303, 24.813\rangle$ or $\langle 2.303, 24.814\rangle$.

$3 : \begin{cases} 1 : x''(4) \\ 1 : \frac{dy}{dt} \\ 1 : \text{answer} \end{cases}$

\#2 2004B #1

(a) At time $t = 0$:

Speed $= \sqrt{x'(0)^2 + y'(0)^2} = \sqrt{3^2 + 7^2} = \sqrt{58}$

Acceleration vector $= \langle x''(0), y''(0)\rangle = \langle 0, -3\rangle$

$2 : \begin{cases} 1 : \text{speed} \\ 1 : \text{acceleration vector} \end{cases}$

(b) $\frac{dy}{dx} = \frac{y'(0)}{x'(0)} = \frac{7}{3}$

Tangent line is $y = \frac{7}{3}(x - 4) + 1$

$2 : \begin{cases} 1 : \text{slope} \\ 1 : \text{tangent line} \end{cases}$

(c) Distance $= \int_0^3 \sqrt{\left(\sqrt{t^4 + 9}\right)^2 + \left(2e^t + 5e^{-t}\right)^2}\, dt$

$= 45.226$ or 45.227

$3 : \begin{cases} 2 : \text{distance integral} \\ \quad \langle -1\rangle \text{ each integrand error} \\ \quad \langle -1\rangle \text{ error in limits} \\ 1 : \text{answer} \end{cases}$

#3

2005 B #1

(a) $x''(2) = 0,\ y''(2) = -\frac{32}{17} = -1.882$

$a(2) = \langle 0, -1.882 \rangle$

Speed $= \sqrt{12^2 + (\ln(17))^2} = 12.329$ or 12.330

$2: \begin{cases} 1: \text{acceleration vector} \\ 1: \text{speed} \end{cases}$

(b) $y(t) = y(0) + \int_0^t \ln\left(1 + (u-4)^4\right) du$

$y(2) = 5 + \int_0^2 \ln\left(1 + (u-4)^4\right) du = 13.671$

$3: \begin{cases} 1: \int_0^2 \ln\left(1 + (u-4)^4\right) du \\ 1: \text{handles initial condition} \\ 1: \text{answer} \end{cases}$

(c) At $t = 2$, slope $= \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}} = \dfrac{\ln(17)}{12} = 0.236$

$y - 13.671 = 0.236(x - 3)$

$2: \begin{cases} 1: \text{slope} \\ 1: \text{equation} \end{cases}$

(d) $x'(t) = 0$ if $t = 0, 4$

$y'(t) = 0$ if $t = 4$

$t = 4$

$2: \begin{cases} 1: \text{reason} \\ 1: \text{answer} \end{cases}$

#4

2006 B#2

(a) $\dfrac{dy}{dx} = \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}} = \dfrac{\sec\left(e^{-t}\right)}{\tan\left(e^{-t}\right)} = \dfrac{1}{\sin\left(e^{-t}\right)}$

$\left.\dfrac{dy}{dx}\right|_{(2,-3)} = \dfrac{1}{\sin\left(e^{-1}\right)} = 2.780$ or 2.781

$y + 3 = \dfrac{1}{\sin\left(e^{-1}\right)}(x - 2)$

$2: \begin{cases} 1: \left.\frac{dy}{dx}\right|_{(2,-3)} \\ 1: \text{equation of tangent line} \end{cases}$

(b) $x''(1) = -0.42253,\ y''(1) = -0.15196$

$a(1) = \langle -0.423, -0.152 \rangle$ or $\langle -0.422, -0.151 \rangle$.

speed $= \sqrt{\left(\sec\left(e^{-1}\right)\right)^2 + \left(\tan\left(e^{-1}\right)\right)^2} = 1.138$ or 1.139

$2: \begin{cases} 1: \text{acceleration vector} \\ 1: \text{speed} \end{cases}$

(c) $\int_1^2 \sqrt{(x'(t))^2 + (y'(t))^2}\, dt = 1.059$

$2: \begin{cases} 1: \text{integral} \\ 1: \text{answer} \end{cases}$

(d) $x(0) = x(1) - \int_0^1 x'(t)\, dt = 2 - 0.775553 > 0$

The particle starts to the right of the y-axis.
Since $x'(t) > 0$ for all $t \geq 0$, the object is always moving to the right and thus is never on the y-axis.

$3: \begin{cases} 1: x(0) \text{ expression} \\ 1: x'(t) > 0 \\ 1: \text{conclusion and reason} \end{cases}$

#5 2006 #3

(a) $a(2) = \langle 0.395 \text{ or } 0.396, -0.741 \text{ or } -0.740\rangle$

Speed $= \sqrt{x'(2)^2 + y'(2)^2} = 1.207$ or 1.208

$2 : \begin{cases} 1 : \text{acceleration} \\ 1 : \text{speed} \end{cases}$

(b) $\sin^{-1}\left(1 - 2e^{-t}\right) = 0$

$1 - 2e^{-t} = 0$

$t = \ln 2 = 0.693$ and $\frac{dy}{dt} \neq 0$ when $t = \ln 2$

$2 : \begin{cases} 1 : x'(t) = 0 \\ 1 : \text{answer} \end{cases}$

(c) $m(t) = \frac{4t}{1+t^3} \cdot \frac{1}{\sin^{-1}\left(1 - 2e^{-t}\right)}$

$$\lim_{t\to\infty} m(t) = \lim_{t\to\infty}\left(\frac{4t}{1+t^3} \cdot \frac{1}{\sin^{-1}\left(1-2e^{-t}\right)}\right)$$

$$= 0\left(\frac{1}{\sin^{-1}(1)}\right) = 0$$

$2 : \begin{cases} 1 : m(t) \\ 1 : \text{limit value} \end{cases}$

(d) Since $\lim_{t\to\infty} x(t) = \infty$,

$c = \lim_{t\to\infty} y(t) = -3 + \int_2^\infty \frac{4t}{1+t^3}\, dt$

$3 : \begin{cases} 1\text{: integrand} \\ 1\text{: limits} \\ 1\text{: initial value consistent} \\ \quad \text{with lower limit} \end{cases}$

#6 2007B #2

(a) Speed $= \sqrt{x'(4)^2 + y'(4)^2} = 2.912$

1 : speed at $t = 4$

(b) Distance $= \int_0^4 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt = 6.423$

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(c) $x(4) = x(0) + \int_0^4 x'(t)dt$

$= -3 + 2.10794 = -0.892$

$3 : \begin{cases} 2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{uses } x(0) = -3 \end{cases} \\ 1 : \text{answer} \end{cases}$

(d) The slope is 2, so $\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = 2$, or $\ln\left(t^2+1\right) = 2\arctan\left(\frac{t}{1+t}\right)$.

Since $t > 0$, $t = 1.35766$. At this time, the acceleration is $\langle x''(t), y''(t)\rangle\big|_{t=1.35766} = \langle 0.135, 0.955\rangle$.

$3 : \begin{cases} 1 : \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = 2 \\ 1 : t\text{-value} \\ 1 : \text{values for } x'' \text{ and } y'' \end{cases}$

#7

2008B #1

(a) $a(4) = \langle x''(4), y''(4) \rangle = \langle 0.433, -11.872 \rangle$

1 : answer

(b) $y(0) = 5 + \int_4^0 3\cos\left(\frac{t^2}{2}\right) dt = 1.600$ or 1.601

$3 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{uses } y(4) = 5 \\ 1 : \text{answer} \end{cases}$

(c) Speed $= \sqrt{(x'(t))^2 + (y'(t))^2}$

$$= \sqrt{3t + 9\cos^2\left(\frac{t^2}{2}\right)} = 3.5$$

The particle first reaches this speed when $t = 2.225$ or 2.226.

$3 : \begin{cases} 1 : \text{expression for speed} \\ 1 : \text{equation} \\ 1 : \text{answer} \end{cases}$

(d) $\int_0^4 \sqrt{3t + 9\cos^2\left(\frac{t^2}{2}\right)}\, dt = 13.182$

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$