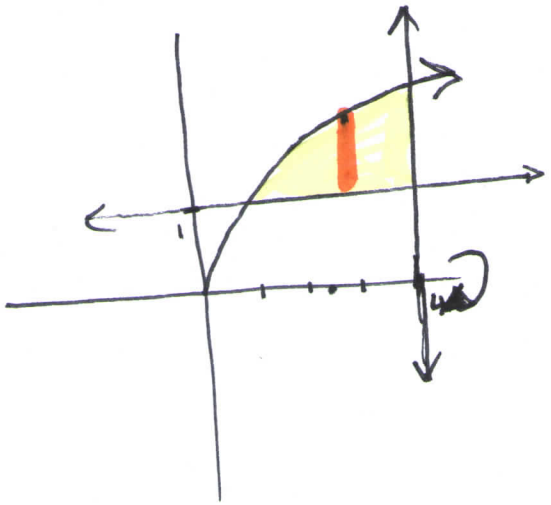
Volume – the Washer method

washer – a disk with a hole in it

$$V = \pi \int_a^b \left[\underbrace{R^2(x)}_{\text{outer radius}} - \underbrace{r^2(x)}_{\text{inner radius}} \right] dx$$

Find the volume.

EX 1 $y = \sqrt{x}$, $y = 1$, $x = 4$ revolve around x-axis



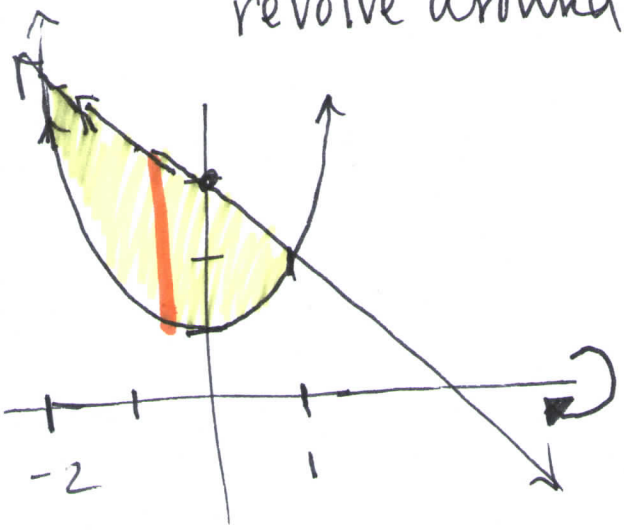
$$V = \pi \int_1^4 (\sqrt{x})^2 - (1)^2 \, dx$$

$$\pi \left[\frac{1}{2}x^2 - x + C \Big|_1^4 \right]$$

$$\pi \left[8 - 4 - \left(\frac{1}{2} - 1 \right) \right]$$

$$\boxed{\frac{9\pi}{2}}$$

Ex2 $y = x^2 + 1$, $y = -x + 3$
revolve around x-axis



$x^2 + 1 = -x + 3$

$x^2 + x - 2 = 0$

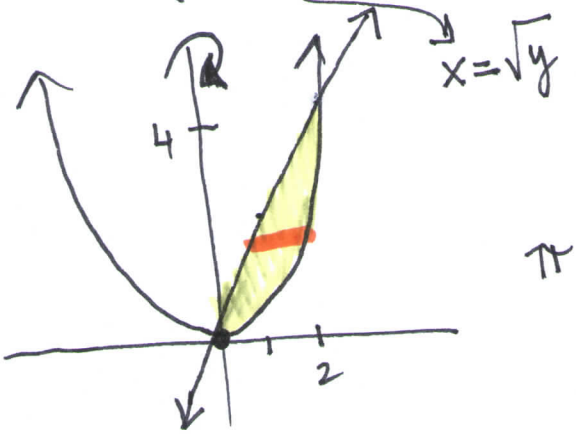
$(x+2)(x-1) = 0$

$x = -2, 1$

$$\pi \int_{-2}^{1} \left[(-x+3)^2 - (x^2+1)^2 \right] dx$$

$$\frac{117}{5}\pi$$

Ex3 $y = x^2$, $y = 2x$ $\rightarrow x = \frac{y}{2}$ revolve about the y-axis



$$\pi \int_{0}^{4} \left(\sqrt{y}\right)^2 - \left(\frac{y}{2}\right)^2 dy = \frac{8\pi}{3}$$

$x^2 = 2x$

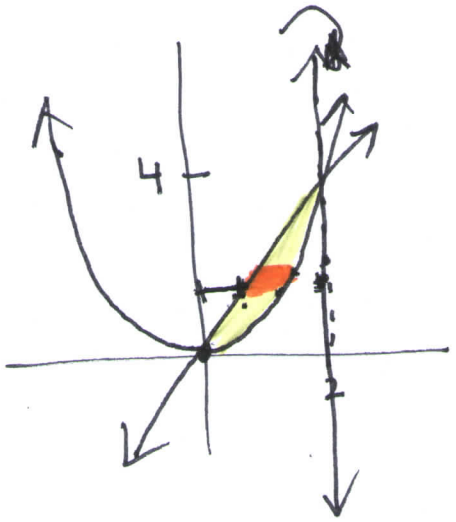
$x^2 - 2x = 0$

$x(x-2) = 0$

$x = 0, x = 2$

$y = 0 \quad y = 4$

EX 4 $y = x^2$ $x = \sqrt{y}$ $y = 2x$ $x = \frac{y}{2}$ revolve around $x = 2$



$$\pi \int_0^4 \left[\left(2 - \frac{y}{2}\right)^2 - \left(2 - \sqrt{y}\right)^2 \right] dy$$

$\frac{8\pi}{3}$

8.378