1995 – BC4

4. Let f be a function that has derivatives of all orders for all real numbers.
Assume $f(1)=3$, $f'(1)=-2$, $f''(1)=2$, and $f'''(1)=4$

(a) Write the second-degree Taylor polynomial for f about x = 1 and use it to approximate $f(0.7)$.
(b) Write the third-degree Taylor polynomial for f about x = 1 and use it to approximate $f(1.2)$.
(c) Write the second-degree Taylor polynomial for f', the derivative of f, about x = 1 and use it to approximate $f'(1.2)$.

(a) $f(x)=f(a)+f'(a)(x-a)+\frac{f''(a)}{2!}(x-a)^2+\frac{f'''(a)}{3!}(x-a)^3+\cdots$

$f(x)\approx f(1)+f'(1)(x-1)+\frac{f''(1)}{2!}(x-1)^2$

$f(x)\approx 3-2(x-1)+\frac{2}{2}(x-1)^2$

$f(0.7)\approx 3+0.6+0.09=3.69$

$\boxed{f(0.7)\approx 3.69}$

(b) $f(x)\approx f(1)+f'(1)(x-1)+\frac{f''(1)}{2!}(x-1)^2+\frac{f'''(1)}{3!}(x-1)^3$

$f(x)\approx 3-2(x-1)+(x-1)^2+\frac{4}{3!}(x-1)^3$

$f(1.2)\approx 3-0.4+0.04+\frac{2}{3}(0.008)=2.645\overline{3}$

$\boxed{f(1.2)\approx 2.645}$

(c) $f'(x)\approx -2+2(x-1)+2(x-1)^2$

$f'(1.2)\approx -2+2(.2)+2(.2)^2$

$\boxed{f'(1.2)\approx -1.52}$