2. Let $P(x) = 7 - 3(x-4) + 5(x-4)^2 - 2(x-4)^3 + 6(x-4)^4$ be the fourth-degree Taylor polynomial for the function f about 4. Assume f has derivatives of all orders for all real numbers.

   (a) Find $f(4)$ and $f'''(4)$.

   (b) Write the second-degree Taylor polynomial for f' about 4 and use it to approximate $f'(4.3)$.

   (c) Write the fourth-degree Taylor polynomial for $g(x) = \int_4^x f(t)\,dt$ about 4.

   (d) Can $f(3)$ be determined from the information given? Justify your answer.

(a) $f(4) = P(4) = 7$

$$\frac{f'''(4)}{3!} = -2, \quad f'''(4) = -12$$

$2 \begin{cases} 1:\ f(4) = 7 \\ 1:\ f'''(4) = -12 \end{cases}$

(b) $P_3(x) = 7 - 3(x-4) + 5(x-4)^2 - 2(x-4)^3$

$P_3'(x) = -3 + 10(x-4) - 6(x-4)^2$

$f'(4.3) \approx -3 + 10(0.3) - 6(0.3)^2 = -0.54$

3:
- 2: $P_3'(x)$
  - $<-1>$ for each incorrect term
  - $<-1>$ for each additional term or $+\ldots$
  - 0/2 if degree < 2
- 1: evaluation (only if degree 1, 2, 3)

(c) $P_4(g,x) = \int_4^x P_3(t)\,dt$

$$= \int_4^x \left[7 - 3(t-4) + 5(t-4)^2 - 2(t-4)^3\right] dt$$

$$= 7(x-4) - \frac{3}{2}(x-4)^2 + \frac{5}{3}(x-4)^3 - \frac{1}{2}(x-4)^4$$

2:
- 1: $P_4(g,x) = \int_4^x P_3(t)\,dt$ or $P_4(g,x) = \int_4^x P(t)\,dt$ (ignore $+\ldots$)
- 1: answer

(d) No. The information given provides values for $f(4)$, $f'(4)$, $f''(4)$, $f'''(4)$, and $f^{(4)}(4)$ only.

$2 \begin{cases} 1:\ \text{answer} \\ 1:\ \text{reason} \end{cases}$