Pre-Calculus

Notes (4.1)---Angles & Their Measure

Angles are measured a couple of different ways. The first unit of measurement is a degree in which 360° (degrees) is equal to one revolution. Most likely the reason why we use 360 is from the Babylonians, whose year is based on 360 days. Another unit of measurement for angles is radians. In radians, 2π is equal to one revolution.

$$\frac{180^\circ}{180^\circ} = \frac{\pi}{180^\circ} \qquad 1^\circ = \frac{\pi}{180^\circ}\text{rad.}$$

So a conversion between radians and degrees $360° = 2\pi$ or $\frac{180°}{\pi} = \frac{\pi}{\pi}$ $\qquad \frac{180^\circ}{\pi} = 1$ radian

When converting from degrees to radians:	When converting from radians to degrees:
Multiply your degrees by $\frac{\pi}{180^\circ}$	Multiply your radians by $\frac{180^\circ}{\pi}$

Ex 1) a) Convert 120° to radians

$$120^\circ \cdot \frac{\pi}{180^\circ} = \frac{120\pi}{180} = \frac{2\pi}{3} \text{ radians}$$

b) Convert $\frac{4\pi}{3}$ into degrees.

$$\frac{4\cancel{\pi}}{3} \cdot \frac{180^\circ}{\cancel{\pi}} = \frac{720^\circ}{3} = 240^\circ$$

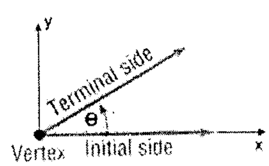
c) How many radians are in 90°?

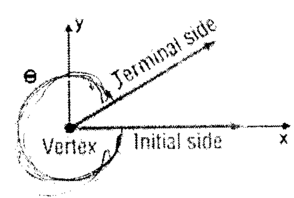
$$90^\circ \cdot \frac{\pi}{180^\circ} = \frac{\pi}{2}$$

d) How many degrees are in $\frac{\pi}{3}$ radians?

$$\frac{\pi}{3} \cdot \frac{180^\circ}{\pi} = 60^\circ$$

We will use θ (theta) to represent an angle's measurement. In the figure below it describes how you know if an angle is positive or negative.





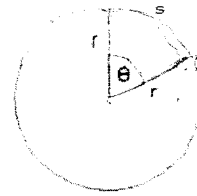
Counterclockwise rotation
Positive angle

Clockwise rotation
Negative angle

The vertex of the angle is at the origin of a rectangular coordinate system. The positive x-axis is always where an angle is measured from, and this is called the initial side. An angle drawn this way is said to be in **standard form**.

An angle that goes counterclockwise is always positive, and clockwise angles are negative.

***Arc Length Formula**(Radian Measure)*



The length of the arc between the two lines (intercepted arc) shown with θ. The equation:

$$s = r\theta$$

where S is the arc length, r is the radius, and θ MUST be measured in RADIANS!
The θ is also called the central angle.

Ex 2) a) Find the length of an arc intercepted by a central angle of ½ radian in a circle with radius 5in.

$\theta = \frac{1}{2}$

$r = 5$

$S = r\theta$

$S = 5 \cdot \frac{1}{2} = \boxed{\frac{5}{2} \text{ in}}$

b) Find the radian measure of a central angle intercepting an arc length 18 meters in a circle of radius 3 meters.

θ

$S = 18$

$r = 3$

$S = r\theta$

$18 = 3 \cdot \theta$

$\boxed{\theta = 6 \text{ radians}}$

c) Find the length of an arc intercepted by a central angle of $\frac{120}{\pi}$ radians ~~degrees~~ in a circle with a radius of 7ft.

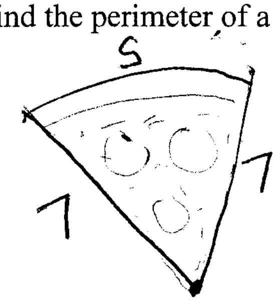
θ

$r = 7$

$S = r\theta$

$S = 7 \cdot \frac{120}{\pi} = \boxed{\frac{840}{\pi} \text{ ft}}$

d) Find the perimeter of a 60° slice of a large (7 in. radius) pizza.



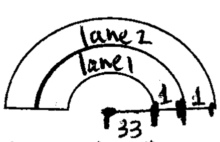
$S = r\theta$

$S = 7 \cdot \frac{\pi}{3} = \boxed{\frac{7\pi}{3} \text{ in}}$

$60° \cdot \frac{\pi}{180°} = \frac{60\pi}{180} = \frac{\pi}{3}$

$\text{perimeter} = 7 + 7 + \frac{7\pi}{3} = \boxed{14 + \frac{7\pi}{3} \text{ in}}$

e) The running lanes at Emery Sears track at Bluffton College are 1m wide. The inner radius of lane 1 is 33 meters. If the inner radius of lane 2 is 34 meters, how much longer is lane 2 than lane 1?



Looking at the 1st turn in the

.34

Lane 1

$S = 33\pi$

Lane 2

$S = 34\pi$

$\boxed{\pi \text{ meters}}$

3.14