



### Angular Speed ( $\omega$ )

Angular speed is the rate at which an object changes its angle  $\theta$  (in radians) in time  $t$ . The formula is:  $\omega = \frac{\theta}{t}$  [Units: radians per period of time]

### Linear Speed ( $v$ )

The speed at which a point on the circle is moving. is measured by what arc length is traveled in time  $t$ .

The formula is:  $v = r\omega$  where  $\omega$  is measured in radians per unit of time.

↑  
radius

$s = r\theta$   $v = r\frac{\theta}{t} = r\omega$    
 (angular speed)

Ex 4) Find  $\omega$  for the following situations:

a) A line from the center to the edge of a CD revolving 300 times per minute.

$$\omega = \frac{300 \text{ rev}}{1 \text{ min}} \cdot \frac{2\pi \text{ rad}}{1 \text{ rev}} = \boxed{600\pi \text{ rad/min}}$$

b) A second hand on a clock.

$$\omega = \frac{1 \text{ rev}}{60 \text{ sec}} \cdot \frac{2\pi \text{ rad}}{1 \text{ rev}} = \frac{2\pi}{60} \text{ rad/sec} = \boxed{\frac{\pi}{30} \text{ rad/sec}}$$

Ex 5) Find  $v$  for each of the following:

a) The tip of the second hand on a clock, if the hand is 28mm long.

$$v = 28 \left( \frac{\pi}{30} \right) = \frac{28\pi}{30} \text{ mm/sec} = \boxed{\frac{14\pi}{15} \text{ mm/sec}}$$

b) A point of tread on a tire with radius 18cm, rotating at 35 times per minute.

$$\omega = \frac{35 \text{ rev}}{1 \text{ min}} \cdot \frac{2\pi \text{ rad}}{1 \text{ rev}} = 70\pi \text{ rad/min}$$

$$v = 18(70\pi) = \boxed{1260\pi \text{ cm/min}}$$

Ex 4) a) A pulley has a radius of 15 cm. Suppose it takes 25 sec for 75 cm of belt to go around the pulley. Find the linear speed of the belt in centimeters per second. Find the angular speed of the pulley in radians per second.

$r = 15 \text{ cm}$

$$v = \frac{75 \text{ cm}}{25 \text{ sec}} = 3 \text{ cm/sec}$$

$$v = r \cdot \omega$$

$$3 = 15 \cdot \omega$$

$$\omega = \frac{3}{15} \text{ rad/sec} = \boxed{\frac{1}{5} \text{ rad/sec}}$$

b) A thread is being pulled off a spool at the rate of 75 cm per sec. Find the radius of the spool if it makes 110 revolutions per min.

$$v = 75 \text{ cm/sec} \quad \frac{75 \text{ cm}}{1 \text{ sec}} \cdot \frac{60 \text{ sec}}{1 \text{ min}} = 4500 \text{ cm/min}$$

$$\omega = \frac{110 \text{ rev}}{1 \text{ min}} \cdot \frac{2\pi \text{ rad}}{1 \text{ rev}} = 220\pi \text{ rad/min}$$

$$v = r\omega$$

$$4500 \text{ cm/min} = r (220\pi \text{ rad/min})$$

$$r = \frac{4500}{220\pi} = \boxed{\frac{225}{11\pi} \text{ cm}}$$

c) A circular gear rotates at a rate of 75 rpm (revolutions per minute). What is the angular speed (in radians per minute)? What is the linear speed of a point on the gear 3mm from the center?

$$\omega = \frac{75 \text{ rev}}{1 \text{ min}} \cdot \frac{2\pi \text{ rad}}{1 \text{ rev}} = \boxed{150\pi \text{ rad/min}}$$

$$v = 3(150\pi) = \boxed{450\pi \text{ mm/min}}$$

d) Yank Hardy pulls the cord on his ~~power~~ mower. In order for the engine to start, the pulley must turn at 180 revolutions per minute. The pulley has a radius of 0.2 feet.

i) At how many radians per ~~hour~~<sup>minute</sup> must the pulley turn?

$$\omega = \frac{180 \text{ rev}}{1 \text{ min}} \cdot \frac{2\pi \text{ rad}}{1 \text{ rev}} = \boxed{360\pi \text{ rad/min}}$$

ii) When Yank pulls the cord to start the mower, what is the ~~angular~~<sup>linear</sup> velocity of the center of the pulley?

$$v = 0.2(360\pi) = \boxed{72\pi \text{ ft/min}}$$

convert to m.p.h.

$$\frac{72\pi \text{ ft}}{1 \text{ min}} \cdot \frac{60 \text{ min}}{1 \text{ hr}} \cdot \frac{1 \text{ mi}}{5280 \text{ ft}}$$

$$\frac{9\pi}{11} \text{ mph} \approx 2.57 \text{ mph}$$

e) A ceiling fan with 16 in. blades rotates at 45 rpm.

i) Find the angular speed of the fan in rad/min.

$$\omega = \frac{45 \text{ rev}}{1 \text{ min}} \cdot \frac{2\pi \text{ rad}}{1 \text{ rev}} = \boxed{90\pi \text{ rad/min}}$$

ii) Find the linear speed of the tips of the blades in in/min.

$$v = 16(90\pi) = \boxed{1440\pi \text{ in/min}}$$