

Find the average value of the each function over the given interval. Then find the values(s) of c guaranteed by the Mean Value Theorem for Integrals for the function over the given interval.

1) $f(x) = -x + 2, [-2, 2]$

$$f(c) = 2 ; c = 0$$

2) $f(x) = -x^2 - 8x - 17, [-6, -3]$

$$f(c) = -2 ; c = -5, c = -3$$

3) $f(x) = 4 - x^2, [-2, 2]$

$$f(c) = \frac{8}{3} ; c = \pm \frac{2}{\sqrt{3}}$$

4) $f(x) = \sin x, [0, \pi]$

$$f(c) = \frac{2}{\pi} ; c = .690 \notin [2.45]$$

In problems 5-7, (a) integrate to find F as a function of x and (b) demonstrate the Second Fundamental Theorem of Calculus by differentiating the result in part (a).

5) $F(x) = \int_0^x (t+2) dt$

$$a) F(x) = \frac{1}{2}x^2 + 2x \quad b) F'(x) = x+2$$

6) $F(x) = \int_0^x \sqrt[3]{t} dt$

$$a) F(x) = \frac{3}{4}x^{\frac{4}{3}} \quad b) F'(x) = x^{\frac{1}{3}}$$

7) $F(x) = \int_{\pi/4}^x \sec^2 t dt$

$$a) F(x) = \tan x - 1 \quad b) F'(x) = \sec^2 x$$

In exercises 8-10, use the Second Fundamental Theorem of Calculus to find $F'(x)$.

8) $F(x) = \int_{-2}^x (t^2 - 2t) dt$

$$x^2 - 2x$$

9) $F(x) = \int_{-1}^x \sqrt{t^4 + 1} dt$

$$\sqrt{x^4 + 1}$$

10) $F(x) = \int_0^x t \cos t dt$

$$x \cos x$$

In exercises 11-13, find $F'(x)$.

11) $F(x) = \int_x^{x+2} (4t + 1) dt$

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12) $F(x) = \int_0^{\sin x} \sqrt{t} dt$

$\sqrt{\sin x} \cdot \cos x$

13) $F(x) = \int_0^{x^3} \sin t^2 dt$

$3x^2 \cdot \sin(x^6)$

2nd FTC

1. $f(x) = -x+2 \quad [-2, 2]$

$$f(c) = \frac{1}{2-(-2)} \int_{-2}^2 (-x+2) dx = \frac{1}{4} \left[-\frac{1}{2}x^2 + 2x + C \right]_{-2}^2$$

$$= \frac{1}{4} \left[-2 + 4 + C - (-2 - 4 + C) \right] = \frac{1}{4} \cdot 8 = \boxed{2}$$

$$-x+2 = 2$$

$$x = 0 \quad \boxed{c=0}$$

2. $f(x) = -x^2 - 8x - 17 \quad [-6, -3]$

$$f(c) = \frac{1}{-3-(-6)} \int_{-6}^{-3} (-x^2 - 8x - 17) dx = \frac{1}{3} \left[-\frac{1}{3}x^3 - 4x^2 - 17x + C \right]_{-6}^{-3}$$

$$= \frac{1}{3} \left[9 - 36 + 51 + C - (72 - 144 + 102 + C) \right] = \frac{1}{3} \cdot -6 = \boxed{-2}$$

$$-x^2 - 8x - 17 = -2$$

$$0 = x^2 + 8x + 15$$

$$0 = (x+5)(x+3)$$

$$x = -5 \quad x = -3$$

$$\boxed{c=-5 \quad c=-3}$$

3. $f(x) = 4-x^2 \quad [-2, 2]$

$$f(c) = \frac{1}{2-(-2)} \int_{-2}^2 (4-x^2) dx = \frac{1}{4} \left[4x - \frac{1}{3}x^3 + C \right]_{-2}^2$$

$$= \frac{1}{4} \left[8 - \frac{8}{3} + C - \left(-8 + \frac{8}{3} + C \right) \right] = \frac{1}{4} \cdot \frac{32}{3} = \boxed{\frac{8}{3}}$$

$$4-x^2 = \frac{8}{3}$$

$$-x^2 = -\frac{4}{3}$$

$$x = \pm \sqrt{\frac{4}{3}} = \pm \frac{2}{\sqrt{3}}$$

$$\boxed{c = \pm \frac{2}{\sqrt{3}}}$$

4. $f(x) = \sin x \quad [0, \pi]$

$$f(c) = \frac{1}{\pi-0} \int_0^\pi \sin x dx = \frac{1}{\pi} (-\cos x) \Big|_0^\pi = \frac{1}{\pi} \left[-\cos \pi + C - (-\cos 0 + C) \right]$$

$$= \frac{1}{\pi} [1+C+1-C] = \boxed{\frac{2}{\pi}}$$

$$\sin x = \frac{2}{\pi} \quad (\text{use calc})$$

$$C = .690 \approx 2.451$$

$$5. \text{ a) } F(x) = \int_0^x (t+2) dt = \frac{1}{2}t^2 + 2t + C \Big|_0^x = \frac{1}{2}x^2 + 2x + C - (0+0+C) \\ = \frac{1}{2}x^2 + 2x$$

$$\text{b) } F'(x) = x+2$$

$$6. \text{ a) } F(x) = \int_0^x \sqrt[3]{t} dt = \frac{3}{4}t^{\frac{4}{3}} + C \Big|_0^x = \frac{3}{4}x^{\frac{4}{3}} + C - (0+C) = \frac{3}{4}x^{\frac{4}{3}}$$

$$\text{b) } F'(x) = \frac{3}{4} \cdot \frac{4}{3}x^{\frac{1}{3}} = x^{\frac{1}{3}}$$

$$7. \text{ a) } F(x) = \int_{\pi/4}^x \sec^2 t dt = \tan t + C \Big|_{\pi/4}^x = \tan x + C - (\tan \frac{\pi}{4} + C) \\ = \tan x - 1$$

$$\text{b) } F'(x) = \sec^2 x$$

$$8. \frac{d}{dx} \int_{-2}^x (t^2 - 2t) dt = x^2 - 2x$$

$$9. \frac{d}{dx} \int_{-1}^x \sqrt{t^4 + 1} dt = \sqrt{x^4 + 1}$$

$$10. \frac{d}{dx} \int_0^x t \cos t dt = x \cos x$$

$$11. \frac{d}{dx} \int_x^{x+2} (4t+1) dt = \frac{d}{dx} \left[\int_x^0 (4t+1) dt + \int_0^{x+2} (4t+1) dt \right] \\ = \frac{d}{dx} \left[\int_0^x (4t+1) dt + \int_0^{x+2} (4t+1) dt \right] = -(4x+1) + (4(x+2)+1) \cdot 1 \\ = -4x - 1 + 4x + 8 + 1 = \boxed{8}$$

$$12. F'(x) = \sqrt{\sin x} \cdot \cos x$$

$$13. F'(x) = \sin(x^3)^2 \cdot 3x^2 = 3x^2 \sin(x^6)$$