KEY

WHAT DO YOU CALL A LAWYER WHO WORKS WEEKENDS AS A LUMBERJACK AND EVENINGS IN A JAZZ CLUB?

Laws of

$$\log(ab) = \log(a) + \log(b) \qquad \log\frac{a}{b} = \log(a) - \log(b)$$
$$\log(a^n) = n\log(a)$$

Logarithms

Match each logarithmic expression with a correct expanded form.

1) $\log(xy)$ G	2) $\log(x + y)$ O	3) $\log(x^2y)$ E	4) $\log(xy)^2$ A
5) $\log\frac{x}{y}$ K	6) $\log(\sqrt{xy})$ H	7) $\log(2x\sqrt{y})$ I	8) $\log\frac{x^2}{2y}$ L

Expanded Form

A. $2\log(x) + 2\log(y)$	E. $2\log(x) + \log(y)$
H. $\frac{1}{2}\log(x) + \frac{1}{2}\log(y)$	I. $\log(2) + \log(x) + \frac{1}{2}\log(y)$
G. $\log(x) + \log(y)$	K. $\log(x) - \log(y)$
L. $2\log(x) - (\log(2) + \log(y))$	M. $\log(y) - \log(x)$
N. $2\log(x) - \log(2) + \log(y)$	O. can't expand

Find the approximate value of each logarithm.
Given: **log5 ≈ 0.699**, **log2 ≈ 0.301**, and **log3 ≈0.477.**

S 9) $\log4 = \log2^2 = 2\cdot\log2$ .602	M 10) $\log10 = \log(2\cdot5) = \log2+\log5$ 1.000	N 11) $\log81 = \log3^4 = 4\cdot\log3$ 1.908	T 12) $\log\frac{1}{5} = \log5^{-1} = -\log5$ -.699
R 13) $\log30 = \log(2\cdot3\cdot5) = \log2+\log3+\log5$ 1.477	W 14) $\log\frac{2}{3} = \log2-\log3$ -.176	Y 15) $\log18 = \log2\cdot3^2 = \log2 + 2\log3 = 1.255$	F 16) $\log0.6 = \log\frac{3}{5} = \log3 - \log5 = -.222$

Value

F. −0.222	K. 1.431	M. 1.000	N. 1.908	P. −0.778	R. 1.477
S. 0.602	T. −0.699	U. 0.778	W. −0.176	Y. 1.255	Z. 0.222

A	M	A	N	W	H
4	10	4	11	14	6

O	R	E	A	L	L
2	13	3	4	8	8

Y	K	N	O	W	S
15	5	11	2	14	9

H	I	S	L	A	W
6	7	9	8	4	14

S	O	F	L	O	G
9	2	16	8	2	1

G	E	R	R	H	Y	T
1	3	13	13	6	15	12

H	M	S
6	10	9

KEY

eeeeeeeee!

ln (pronounced "L" "N")
is called the **natural logarithm**.
It is the logarithm with a **base of e**.

$$\ln(x) = \log_e(x)$$

Simplify each natural exponential or logarithmic expression and match with its result.

O 1) $e^2 \cdot e^3 = e^5$	N 2) $(e^3)^2 = e^6$	R 3) $(e^6)^{0.5} = e^3$	L 4) $e \cdot e^{-5} = e^{-4} = \frac{1}{e^4}$
P 5) $e^x \cdot e^{-x+2} = e^2$	R 6) $\frac{e^4}{e} = e^3$	S 7) $\frac{e^{-2}}{e^{-3}} = e^1$	S 8) $e^{\ln e} = e$
C 9) $\ln e = 1$	E 10) $\ln e^2 = 2$	D 11) $\ln \frac{1}{e} = -1$	A 12) $\ln 1 = 0$
Y 13) $\ln 0$ not possible	H 14) $-\ln \frac{1}{2} = -\ln 2^{-1} = \ln 2$	T 15) $\ln\sqrt{e} = \ln e^{\frac{1}{2}} = \frac{1}{2}$	E 16) $\frac{1}{2}\ln e^4 = \frac{1}{2} \cdot 4 = 2$
I 17) $\ln 2 + \ln 5 = \ln 10$	W 18) $\ln 8 + \ln 4 = \ln 32 = \ln 2^5 = 5\ln 2$	H 19) $\ln 8 - 2\ln 2$ $\ln 2^3 - 2\ln 2 = 3\ln 2 - 2\ln 2 = \ln 2$	E 20) $e^{\ln 2} = 2$
T 21) $e^{-\ln 2} = e^{\ln 2^{-1}} = \frac{1}{2}$	V 22) $e^{2\ln 3} = e^{\ln 9} = 9$	K 23) $e^{\ln 6 - \ln 2} = e^{\ln 3} = 3$	H 24) $\ln(e^{\ln 2}) = \ln 2$
C 25) $e^2 \cdot e^{-2} = e^0 = 1$	E 26) $\ln(e^{2\ln e}) = \ln e^{\ln e^2} = \ln e^2 = 2$	H 27) $\ln \frac{1}{2} - \ln \frac{1}{4} = \ln 2$	A 28) $\ln 9 - 2\ln 3$ $\ln 9 - \ln 9 = 0$

Answers

A. 0	C. 1	D. –1	E. 2	F. –2	H. ln2	I. ln10
J. –ln2	K. 3	L. e^{-4}	N. e^6	O. e^5	P. e^2	R. e^3
S. e	T. $\frac{1}{2}$	U. $-\frac{1}{2}$	V. 9	W. 5ln2	Y. not possible	

W	H	A	T
18	19	12	15

E	L	L	E	N
26	4	4	26	2

C	R	I	E	D
9	6	17	10	11

W	H	E	N
18	27	16	2

T	H	E
15	24	10

P	O	E	T	R	Y
5	1	10	21	3	13

T	E	A	C	H	E	R
21	20	28	25	14	26	6

S	A	I	D
8	12	17	11

T	O
15	1

S	P	E	A	K
7	5	20	28	23

I	N
17	2

V	E	R	S	E
22	10	3	7	16

KEY

WHY WAS THE CALCULUS STUDENT CONFUSED ABOUT $y = e^x$ AND THE DERIVATIVE OF $y = e^x$?

Find the derivative of each function and match with the result.

1) $y = e^{2x}$ $y' = e^{2x} \cdot 2$	2) $y = 2e^x$ $y' = 2e^x$	3) $y = 2e^{2x} + 2$ $y' = 2 \cdot e^{2x} \cdot 2 = 4e^{2x}$	4) $y = 2e^2$ $y' = 0$
5) $y = \frac{1}{2}e^{4x}$ $y' = \frac{1}{2} \cdot e^{4x} \cdot 4 = 2e^{4x}$	6) $y = -e^{-x}$ $y' = -e^{-x} \cdot -1 = e^{-x}$	7) $y = \frac{1}{2}e^{x^2}$ $y' = \frac{1}{2} \cdot e^{x^2} \cdot 2x = xe^{x^2}$	8) $y = -e^{1/x}$ $y' = -e^{\frac{1}{x}} \cdot -1x^{-2} = \frac{e^{\frac{1}{x}}}{x^2}$
9) $y = 2e^{\sqrt{x}}$ $y' = 2 \cdot e^{\sqrt{x}} \cdot \frac{1}{2}x^{-\frac{1}{2}} = \frac{e^{\sqrt{x}}}{\sqrt{x}}$	10) $y = xe^2$ $y' = e^2$	11) $y = xe^x$ $y' = x \cdot e^x + e^x \cdot 1$ $= e^x(x+1)$	12) $y = \frac{1}{x}e^x$ $y' = \frac{1}{x} \cdot e^x + e^x \cdot -1x^{-2}$ $= \frac{e^x}{x} - \frac{e^x}{x^2}$
13) $y = xe^x - e^x + 2$ $y' = x \cdot e^x + e^x \cdot 1 - e^x = xe^x$		14) $y = x^2e^x - xe^x$ $y' = x^2 \cdot e^x + e^x \cdot 2x - [x \cdot e^x + e^x \cdot 1]$ $= x^2e^x + xe^x + -e^x$	
15) $y = \sqrt{x}\, e^{\sqrt{x}} + 1$ $y' = \sqrt{x} \cdot e^{\sqrt{x}} \cdot \frac{1}{2}x^{-\frac{1}{2}} + e^{\sqrt{x}} \cdot \frac{1}{2}x^{-\frac{1}{2}}$ $= \frac{e^{\sqrt{x}}}{2} + \frac{e^{\sqrt{x}}}{2\sqrt{x}}$		16) $y = (e^x - 1)^2$ $y' = 2(e^x - 1)^1 \cdot e^x$ $= 2e^{2x} - 2e^x$	

Derivatives.

#4 A. $y' = 0$	#10 B. $y' = e^2$	#6 C. $y' = e^{-x}$	#2 D. $y' = 2e^x$	#13 E. $y' = xe^x$
#3 F. $y' = 4e^{2x}$	#1 H. $y' = 2e^{2x}$	#5 I. $y' = 2e^{4x}$	J. $y' = e^x$	K. $y' = e^{x^2}$
#7 L. $y' = xe^{x^2}$	#9 M. $y' = \frac{1}{\sqrt{x}}e^{\sqrt{x}}$	#8 N. $y' = \frac{1}{x^2}e^{1/x}$	#16 O. $y' = 2e^{2x} - 2e^x$	
P. $y' = \sqrt{x}\, e^{\sqrt{x}} + \frac{1}{\sqrt{x}}e^{\sqrt{x}}$	#15 R. $y' = \frac{1}{2}e^{\sqrt{x}} + \frac{1}{2\sqrt{x}}e^{\sqrt{x}}$	#12 S. $y' = \frac{1}{x}e^x - \frac{1}{x^2}e^x$		
#11 T. $y' = xe^x + e^x$	#14 U. $y' = x^2e^x + xe^x - e^x$	Y. $y' = x^2e^x + xe^x + e^x$		

B	E	C	A	U	S	E		H	E		C	O	U	L	D	N	'		T				
10	13	6	4	14	12	13		1	13		6	16	14	7	2	8			11				
D	I	F	F	E	R	E	N	T	I	A	T	E		T	H	E	M						
2	5	3	3	13	15	13	8	11	5	4	11	13		11	1	13	9						

KEY

HOW CAN TWO LUMBERJACKS CHOP TREES IN PERFECT SYNCHRONIZATION?

Find the derivative of each function and match with the result.

1) $y = \ln(x)$ $y' = \frac{1}{x}$	2) $y = \ln(3x)$ $y' = \frac{1}{3x} \cdot 3 = \frac{1}{x}$	3) $y = 3\ln(x)$ $y' = 3 \cdot \frac{1}{x} = \frac{3}{x}$	4) $y = \ln(x^2)$ $y' = \frac{1}{x^2} \cdot 2x = \frac{2}{x}$
5) $y = \frac{1}{2}\ln(2)$ $y' = 0$	6) $y = \ln(e^x) = x$ $y' = 1$	7) $y = \frac{1}{x} = x^{-1}$ $y' = -1x^{-2} = \frac{-1}{x^2}$	8) $y = \ln(x^3)$ $y' = \frac{1}{x^3} \cdot 3x^2 = \frac{3}{x}$
9) $y = \frac{1}{2}\ln(x)$ $y' = \frac{1}{2} \cdot \frac{1}{x} = \frac{1}{2x}$	10) $y = x\ln(e)$ $y' = \ln e = 1$	11) $y = 1 - \ln(\frac{1}{x})$ $y' = -\frac{1}{\frac{1}{x}} \cdot -1x^{-2}$ $= \frac{x}{x^2} = \frac{1}{x}$	12) $y = \ln^2(x) = (\ln x)^2$ $y' = 2 \cdot \ln x \cdot \frac{1}{x}$
13) $y = \ln\sqrt{x}$ $y' = \frac{1}{\sqrt{x}} \cdot \frac{1}{2}x^{-\frac{1}{2}}$ $= \frac{1}{2x}$	14) $y = x\ln(x)$ $y' = x \cdot \frac{1}{x} + (\ln x)(1)$ $= 1 + \ln x$	15) $y = \sqrt{x}\ \ln(x)$ $y' = \sqrt{x} \cdot \frac{1}{x} + (\ln x)(\frac{1}{2}x^{-\frac{1}{2}})$	16) $y = \frac{1}{x}\ln(x)$ $y' = \frac{1}{x} \cdot \frac{1}{x} + (\ln x)(-1x^{-2})$

17) $y = \ln(x^2 - 4)$ $y' = \frac{1}{x^2-4} \cdot 2x$	18) $y = \ln\left(\frac{x}{x+1}\right)$ $y' = \frac{1}{\frac{x}{x+1}} \cdot \frac{(x+1)(1) - x(1)}{(x+1)^2}$ $= \frac{x+1}{x} \cdot \frac{1}{(x+1)^2}$ $= \frac{1}{x(x+1)} = \frac{1}{x^2+x}$	19) $y = \ln(\ln(x))$ $y' = \frac{1}{\ln x} \cdot \frac{1}{x} = \frac{1}{x\ln x}$

Derivatives.

#7 A. $y' = -\frac{1}{x^2}$	B. $y' = \ln(x)$	C. $y' = e^x$	#4 E. $y' = \frac{2}{x}$	#3, 8 G. $y' = \frac{3}{x}$
#1, 2, 11 H. $y' = \frac{1}{x}$	#5 I. $y' = 0$	J. $y' = \frac{1}{x^2}$	K. $y' = x$	#6, 10 L. $y' = 1$
#18 M. $y' = \frac{1}{x^2+x}$	#17 N. $y' = \frac{2x}{x^2-4}$	#19 O. $y' = \frac{1}{x\ln(x)}$	#9, 13 R. $y' = \frac{1}{2x}$	#12 T. $y' = \frac{2\ln(x)}{x}$
#15 U. $y' = \frac{\sqrt{x}}{x} + \frac{1}{2\sqrt{x}}\ln(x)$	#14 W. $y' = \ln(x) + 1$	#16 Y. $y' = \frac{1}{x^2}(1 - \ln(x))$		

W	I	T	H
14	5	12	1

N	A	T	U	R	A	L
17	7	12	15	9	7	6

L	O	G	G	E	R
10	19	3	8	4	9

R	H	Y	T	H	M
13	2	16	12	11	18