

Practice 6.1: Area Between Curves

- 1) Set up the definite integral that gives the area bounded between the curves.

a) $f(x) = x^2 - 6x$
 $g(x) = 0$

b) $f(x) = x^2 - 4x + 3$
 $g(x) = -x^2 + 2x + 3$

c) $f(x) = 3(x^3 - x)$
 $g(x) = 0$

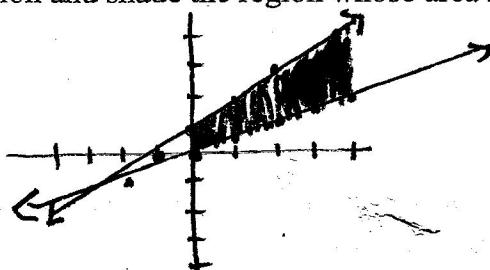
$$\int_0^6 (-x^2 + 6x) dx$$

$$\int_0^3 (-2x^2 + 6x) dx$$

$$2 \int_{-1}^0 3(x^3 - x) dx$$

I used symmetry!

- 2) The integrand of the definite integral $\int_0^4 \left[(x+1) - \frac{x}{2} \right] dx$ is a difference of two functions. Sketch the graph of each function and shade the region whose area is represented by the integral.



- 3) Find the area of the region between the curves $x = 4 - y^2$ and $x = y - 2$ using integration:

- a) with respect to x b) with respect to y .

a) $\int_{-5}^0 [x+2 - (-\sqrt{4-x})] dx + 2 \int_0^4 \sqrt{4-x} dx = \frac{125}{6}$

b) $\int_{-3}^2 [(4-y^2) - (y-2)] dy = \frac{125}{6}$

- 4) Sketch the region bounded by the graphs and find the area of the region.

a) $y = \frac{1}{2}x^3 + 2, y = x + 1, x = 0, x = 2 \quad \int_0^2 \left[\frac{1}{2}x^3 + 2 - (x+1) \right] dx = 2$

b) $f(x) = x^2 + 2x + 1, g(x) = 3x + 3 \quad \int_{-1}^2 [3x+3 - (x^2 + 2x + 1)] dx = 4.5$

c) $f(x) = \sqrt{3x} + 1, g(x) = x + 1 \quad \int_0^3 [\sqrt{3x} + 1 - (x+1)] dx = 1.5$

d) $f(y) = y^2 + 1, g(y) = 0, y = -1, y = 2 \quad \int_{-1}^2 (y^2 + 1 - 0) dy = 6$

e) $f(x) = \cos x, g(x) = 2 - \cos x, 0 \leq x \leq 2\pi \quad \int_0^{2\pi} [(2 - \cos x) - \cos x] dx = 4\pi$

f) $f(x) = xe^{-x^2}, y = 0, 0 \leq x \leq 1 \quad \int_0^1 xe^{-x^2} dx = \frac{e-1}{2e} \approx 0.316 \quad \approx 12.566$