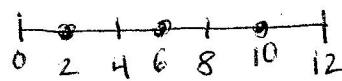


Practice – Area Under a Curve & Riemann Sums

- 1) Let f be a continuous function defined on $[0, 12]$ as shown below. Use LRAM, RRAM, and MRAM to estimate the area under the curve. Use 3 subintervals of equal length.

x	0	2	4	6	8	10	12
$f(x)$	3	7	19	39	67	103	147



$$\text{width} = \frac{12-0}{3} = 4 \quad \text{LRAM: } 4(3+19+67) = \boxed{356}$$

$$= 4 \quad \text{RRAM: } 4(19+67+147) = \boxed{932}$$

$$\text{MRAM: } 4(7+39+103) = \boxed{596}$$

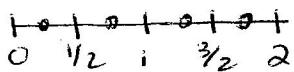
- 2) Use LRAM & RRAM to estimate the area under the curve $y = x^3 + 1$ over the interval $[0, 4]$ using 4 subdivisions of equal length.

$$\text{width} = \frac{4-0}{4} = 1 \quad \text{LRAM: } 1(1+2+9+28) = \boxed{40}$$

$$= 1 \quad \text{RRAM: } 1(2+9+28+65) = \boxed{104}$$

x	y
0	1
1	2
2	9
3	28
4	65

- 3) Use a midpoint Riemann sum w/ 4 subdivisions of equal length to find the approximate value of $\int_0^2 (x^3 + 1) dx$



$$\text{width} = \frac{2-0}{4} = \frac{1}{2}$$

$$\frac{1}{2} \left(\frac{65}{64} + \frac{91}{64} + \frac{189}{64} + \frac{407}{64} \right) \\ = \boxed{5.875}$$

x	$f(x)$
1/4	65/64
3/4	91/64
5/4	189/64
7/4	407/64

- 4) Use LRAM and RRAM to estimate $\int_0^5 f(x) dx$ if $f(x)$ is a continuous function represented by the values in the table below.

x	0	1	3	4.5	5
$f(x)$	5	12	21	40	75

The width is not the same for each interval!

$$\text{LRAM: } 1 \cdot 5 + 2 \cdot 12 + 1.5 \cdot 21 + .5 \cdot 40 = \boxed{80.5}$$

$$\text{RRAM: } 1 \cdot 12 + 2 \cdot 21 + 1.5 \cdot 40 + .5 \cdot 75 = \boxed{151.5}$$