

# WHAT DID THE CALCULUS TEACHER SAY TO HER NERVOUS STUDENTS BEFORE THE QUIZ ON THE CHAIN RULE?

## Derivatives of Composite Functions

If  $y = f(g(x))$

$$y = (1 - 2x^4)^7$$

then  $y' = f'(g(x))g'(x)$

$$y' = 7(1 - 2x^4)^6 \cdot (-8x^3)$$

$$y' = -56x^3(1 - 2x^4)^6$$

Find the derivative  $y'$  of each function.

1) $y = 4x^4 + 12x^2 + 9$ <i>E</i>	2) $y = x^3 + 6x^2 + 12x + 8$ <i>S</i>	3) $y = 1 - 10x^3 + 25x^6$ <i>O</i>
4) $y = (2x^2 + 3)^2$ <i>E</i>	5) $y = (x + 2)^3$ <i>S</i>	6) $y = (1 - 5x^3)^2$ <i>O</i>
7) $y = 3(x^2 + 2x)^2$ <i>R</i>	8) $y = (1 + \sqrt{x})^2$ <i>V</i>	9) $y = \left(1 - \frac{1}{x}\right)^2$ <i>V</i>
10) $y = 2(2x^2 + 3)^5$ <i>C</i>	11) $y = \sqrt{x^2 - 4}$ <i>P</i>	12) $y = -5\sqrt[3]{2-x}$ <i>S</i>
13) $y = \frac{-1}{x^2 - 4}$ <i>M</i>	14) $y = \frac{-1}{\sqrt{x^2 - 4}}$ <i>L</i>	15) $y = -5\sqrt[3]{2-x}$ <i>U</i>

Derivatives.

E. $y' = 8x(2x^2 + 3)$	O. $y' = -30x^2(1 - 5x^3)$	R. $y' = 12x(x+1)(x+2)$	S. $y' = 3(x+2)^2$
C. $y' = 40x(2x^2 + 3)^4$	L. $y' = \frac{x}{(x^2 - 4)^{3/2}}$	M. $y' = \frac{2x}{(x^2 - 4)^2}$	P. $y' = \frac{x}{\sqrt{x^2 - 4}}$
S. $y' = \frac{5}{2\sqrt{2-x}}$	U. $y' = \frac{5}{3(2-x)^{2/3}}$	V. $y' = \frac{1+\sqrt{x}}{\sqrt{x}}$	Y. $y' = \frac{2}{x^2} - \frac{2}{x^3}$

C	O	M	P	O	S	E	Y	O	U	R	S	E	L	V	E	S
10	3	13	11	6	12	4	9	6	15	7	2	1	14	8	4	5

## Chain Rule Puzzle

E ①  $y' = 16x^3 + 24x = 8x(2x^2 + 3)$

S ②  $y' = 3x^2 + 12x + 12 = 3(x^2 + 4x + 4) = 3(x+2)^2$

O ③  $y' = -30x^2 + 150x^5 = -30x^2(1 - 5x^3)$

E ④  $y' = 2(2x^2 + 3)'(4x) = 8x(2x^2 + 3)$

S ⑤  $y' = 3(x+2)^2(1) = 3(x+2)^2$

O ⑥  $y' = 2(1 - 5x^3)'(-15x^2) = -30x^2(1 - 5x^3)$

R ⑦  $y' = 3 \cdot 2(x^2 + 2x)'(2x+2) = 6(x)(x+2) \cdot 2(x+1) = 12x(x+2)(x+1)$

V ⑧  $y = (1 + \sqrt{x})^2 = (1 + x^{1/2})^2$   
 $y' = 2(1 + \sqrt{x})'(1/2x^{-1/2}) = \frac{1 + \sqrt{x}}{\sqrt{x}}$

Y ⑨  $y = (1 - \frac{1}{x})^2 = (1 - x^{-1})^2$   
 $y' = 2(1 - \frac{1}{x})(1x^{-2}) = \frac{2(1 - \frac{1}{x})}{x^2} = \frac{2}{x^2} - \frac{2}{x^3}$

C ⑩  $y' = 2(5)(2x^2 + 3)^4(4x) = 40x(2x^2 + 3)^4$

P ⑪  $y = \sqrt{x^2 - 4} = (x^2 - 4)^{1/2}$   
 $y' = \frac{1}{2}(x^2 - 4)^{-1/2}(2x) = \frac{x}{\sqrt{x^2 - 4}}$

S ⑫  $y = -5(2-x)^{1/2}$   
 $y' = -5 \cdot \frac{1}{2}(2-x)^{-1/2}(-1) = \frac{5}{2\sqrt{2-x}}$

M ⑬  $y = \frac{-1}{x^2 - 4} = -1(x^2 - 4)^{-1}$

$y' = -1 \cdot (-1)(x^2 - 4)^{-2}(2x) = \frac{2x}{(x^2 - 4)^2}$

$$\text{L } 14 \quad y = -1(x^2 - 4)^{-1/2}$$
$$y' = -1\left(-\frac{1}{2}\right)(x^2 - 4)^{-3/2}(2x) = \frac{x}{(x^2 - 4)^{3/2}}$$

$$\text{u } 15 \quad y = -5(2-x)^{1/3}$$
$$y' = -5\left(\frac{1}{3}\right)(2-x)^{-2/3}(-1) = \frac{5}{3(2-x)^{2/3}}$$