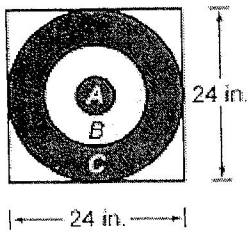
Pre-Calculus
Applications of Conics

1) Bernadine is taking an archery class and decides to practice her skills at home. She attaches the target shown to the right to a bale of hay. The circles on the target are concentric and equally spaced apart.



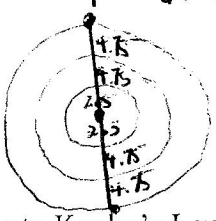
a) If the common center of the circles is located at the origin, write an equation that models the largest circle.

C(0,0) r = 12

$x^2 + y^2 = 144$

b) If the smallest circle is modeled by the equation $x^2 + y^2 = 6.25$, determine the area of the region marked *B*.

$r^2 = 6.25 \Rightarrow r = 2.5$



24 − 5 = 19
÷ 4
4.75
4.75 + 2.5 = 7.25

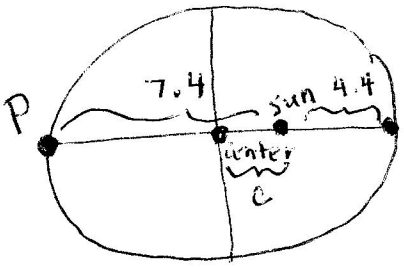
Area of circle = πr^2

Area B − Area A

$(7.25)^2\pi - (2.5)^2\pi$

46.3125π units^2

example 2) According to Kepler's Laws, planets have elliptical orbits, with the sun at one of the foci. The farthest Pluto gets from the sun is 7.4 billion kilometers. The closest it gets to the sun is 4.4 billion kilometers. Determine the equation of Pluto's orbit assuming a center at (0, 0).



major axis = 7.4 + 4.4 = 11.8
÷ 2
5.9

c = 5.9 − 4.4 = 1.5

$\frac{x^2}{(5.9)^2} + \frac{y^2}{32.56} = 1$ $\quad \frac{x^2}{34.81} + \frac{y^2}{32.56} = 1$

$c^2 = a^2 - b^2$

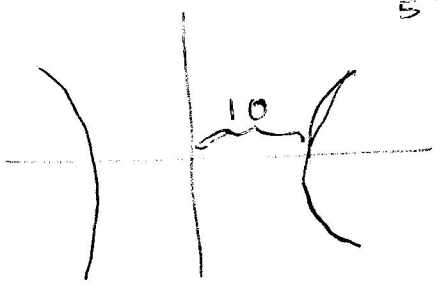
$(1.5)^2 = (5.9)^2 - b^2$

$-32.56 = -b^2$

$32.56 = b^2$

3) Alpha particles are deflected along hyperbolic paths when they are directed towards the nuclei of gold atoms. If an alpha particle gets as close as 10 units to the nucleus along a hyperbolic path with asymptote $y = \frac{2}{5}x$, what is the equation of its path? (Assume the hyperbola opens left and right).

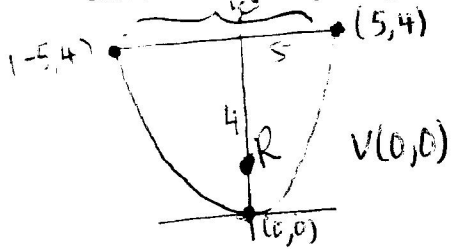
$\frac{2}{5} = \frac{4}{10} \rightarrow b$



$\frac{x^2}{10^2} - \frac{y^2}{b^2} = 1$

$\frac{x^2}{100} - \frac{y^2}{16} = 1$

4) A satellite dish is shaped like a paraboloid of revolution. The signals that emanate from a satellite strike the surface of the dish and are reflected to a single point, where the receiver is located. If the dish is 10 feet across at its opening and is 4 feet deep at its center, at what position should the receiver be placed?



$y = a(x-h)^2 + k$

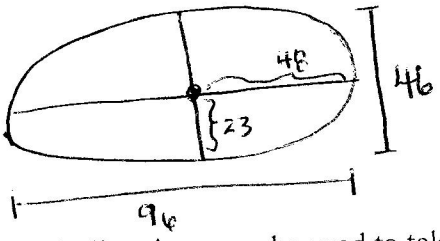
$4 = a(5-0)^2 + 0$

$4 = 25a$

$a = \frac{4}{25}$ $\quad \frac{1}{4p} = \frac{4}{25}$ $\quad 16p = 25$ $\quad p = \frac{25}{16}$

focus at $(0, \frac{25}{16})$

5) The Statuary Hall in the United States Capitol is elliptical. It measures 46 feet wide and 96 feet long. If a person is standing at one focus, her whisper can be heard by a person standing at the other focus. How far apart are the two people?



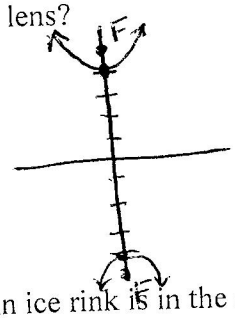
$c^2 = a^2 - b^2$

$c^2 = 48^2 - 23^2$

$c^2 = 1775$

$c = 42.13 \times 2 \Rightarrow$ 84.26 ft apar

6) A hyperbolic mirror can be used to take panoramic photographs. A camera is pointed toward the vertex of the mirror and is positioned so that the lens is at one focus of the mirror. An equation for the cross section of the mirror is $\frac{y^2}{16} - \frac{x^2}{9} = 1$ where x and y are measured in inches. How far from the mirror is the lens?



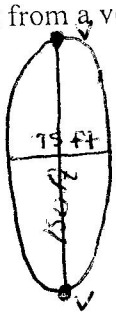
$c^2 = 16 + 9$

$c^2 = 25$

$c = 5$

vertex → focus $= 5 - 4 =$ 1 in

7) An ice rink is in the shape of an ellipse, and is 150 feet long and 75 feet wide. What is the width of the rink 15 feet from a vertex?



$\frac{x^2}{(37.5)^2} + \frac{y^2}{(75)^2} = 1$

15 ft from vertex $\Rightarrow y = 60$

$\frac{x^2}{1406.25} + \frac{60^2}{5625} = 1$

$\frac{x^2}{1406.25} = .36$

$x^2 = 506.25$

$x = 22.5 * 2$

45 ft wi

8) A pizza delivery area can be represented by a circle, and extends to the points $(0, 18)$ and $(-6, 8)$ (these points are on the diameter of this circle). Write an equation for the circle that models this delivery area.

$(0,18)$ $(-6,8)$

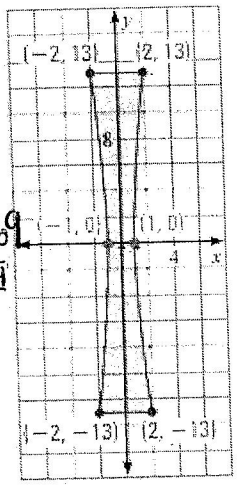
center = midpt. $= (-3, 13)$

diameter $= \sqrt{(0-(-6))^2 + (18-8)^2} = \sqrt{36+100} = \sqrt{136} = 2\sqrt{34}$

radius $= \sqrt{34}$

$(x+3)^2 + (y-13)^2 = 34$

example 9) The diagram at the right shows the hyperbolic cross section of a sculpture located at the Fermi National Accelerator Laboratory in Batavia, Illinois.



a) Write an equation that models the curved sides of the sculpture.

C(0,0)

$\frac{x^2}{1^2} - \frac{y^2}{b^2} = 1$

(2,13) is a pt. on hyperbola

$\frac{(2)^2}{1} - \frac{(13)^2}{b^2} = 1$

$4 - \frac{169}{b^2} = 1$

$\frac{-169}{b^2} = -3$

$-3b^2 = -169$

$b^2 = \frac{169}{3}$

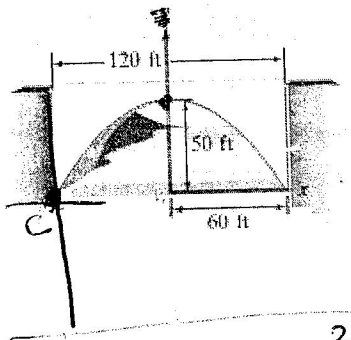
b) At a height of 5 feet, how wide is the sculpture? (Each unit in the coordinate plane represents 1 foot.

$\frac{x^2}{1} - \frac{y^2}{\frac{169}{3}} = 1$

$\frac{x^2}{1} - \frac{5^2}{\frac{169}{3}} = 1$

$x = 1.202 \times 2 = 2.403$ ft

example 10) A concrete bridge is designed with an arch in the shape of a parabola. The road over the bridge is 120 feet long and the maximum height of the arch is 50 feet. Write an equation for the parabolic arch.



(0,0) pt. on parab.

V(60,50)

$y = a(x-h)^2 + k$

$y = a(x-60)^2 + 50$

$0 = a(0-60)^2 + 50$

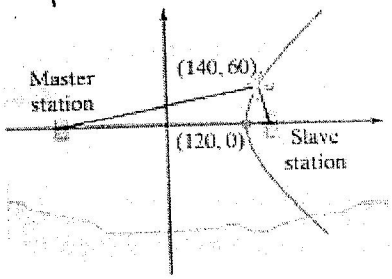
$-50 = 3600a$

$a = \frac{-50}{3600}$

$a = -\frac{1}{72}$

$y = -\frac{1}{72}(x-60)^2 + 50$

11) Long-range navigation (LORAN) is a radio navigation system developed during World War II. The system enables a pilot to guide aircraft by maintaining a constant difference between the aircraft's distances from two fixed points: the master station and the slave station. Determine the equation for the hyperbola depicting this to the right.



C(0,0)

$\frac{x^2}{(120)^2} - \frac{y^2}{b^2} = 1$

$\frac{(140)^2}{(120)^2} - \frac{(60)^2}{b^2} = 1$

$\frac{49}{36} - \frac{3600}{b^2} = 1$

$-\frac{3600}{b^2} = -\frac{13}{36}$

$-13b^2 = -129600$

$b^2 = 9969.23$

$\frac{x^2}{14400} - \frac{y^2}{9969.23} = 1$

OR

$\frac{x^2}{14400} - \frac{y^2}{\frac{129600}{13}} = 1$