AP Calculus AB

Practice Volume: Cross Sections

1) Find the volume of the solid whose base is bounded by the graphs of $y = x + 1$ and $y = x^2 - 1$, with the indicated cross sections taken perpendicular to the x-axis.

a) Squares

$$\int_{-1}^{2} \left(-x^2 + x + 2\right)^2 dx$$

$$\frac{81}{10}$$

b) Rectangles of height 1

$$\int_{-1}^{2} \left(-x^2 + x + 2\right) dx$$

$$\frac{9}{2}$$

2) Find the volume of the solid whose base is bounded by the circle $x^2 + y^2 = 4$ with the indicated cross sections taken perpendicular to the x-axis.

a) Squares

$$\int_{-2}^{2} \left(16 - 4x^2\right) dx$$

$$\frac{128}{3}$$

b) Equilateral triangles

$$\int_{-2}^{2} \sqrt{3}\left(4 - x^2\right) dx$$

$$\frac{32\sqrt{3}}{3}$$

c) Semicircles

$$\int_{-2}^{2} \frac{1}{2}\pi \left(4 - x^2\right) dx$$

$$\frac{16\pi}{3}$$

d) Isosceles right triangles with the hypotenuse on the plane

$$\int_{-2}^{2} \left(4 - x^2\right) dx$$

$$\frac{32}{3}$$

3) The base of a solid is bounded by $y = x^3$, $y = 0$ and $x = 1$. Find the volume of the solid for each of the following cross sections perpendicular to the y-axis.

a) Squares

$$\int_{0}^{1} \left(1 - \sqrt[3]{y}\right)^2 dy$$

$$\frac{1}{10}$$

b) Equilateral triangles

$$\int_{0}^{1} \frac{\sqrt{3}}{4}\left(1 - \sqrt[3]{y}\right)^2 dy$$

$$\frac{\sqrt{3}}{40}$$

c) Semicircles

$$\int_{0}^{1} \frac{\pi}{8}\left(1 - \sqrt[3]{y}\right)^2 dy.$$

$$\frac{\pi}{80}$$