KEY

Calculus AB

Introduction to Curve Sketching (Day 1)

Locate the absolute extrema of the function on the closed interval.

1) $f(x) = -x^2 + 3, \quad [0, 3]$

$f'(x) = -2x$

$-2x = 0$

$x = 0$

x	f(x)
0	3
3	-6

(3, -6) abs. min

(0, 3) abs. max

2) $y = 3x^{\frac{2}{3}} - 2x, \quad [-1, 1]$

$y' = 2x^{-1/3} - 2$

$\frac{2}{\sqrt[3]{x}} - 2 = 0$

$\frac{2}{\sqrt[3]{x}} = 2$

$2\sqrt[3]{x} = 2$

$\sqrt[3]{x} = 1$

$x = 1$

(0,0) abs. min

(-1, 5) abs max

x	f(x)
0	0
1	1
-1	5

Find the critical numbers of f (if any). Find the open intervals on which the function is increasing or decreasing and locate the relative extrema.

3) $f(x) = 2x^3 + 3x^2 - 12x$

$f'(x) = 6x^2 + 6x - 12$

$6(x^2 + x - 2) = 0$

$6(x+2)(x-1) = 0$

$x = -2, 1$

$f'(x)$: + − +

x: -2 1

critical #: x = -2, 1

incr $(-\infty, -2)$

$(1, \infty)$

decr $(-2, 1)$

rel max $(-2, 20)$

rel min $(1, -7)$

4) $f(x) = x^2(3 - x)$

$f'(x) = x^2(-1) + (3-x)(2x)$

$= -x^2 + 6x - 2x^2 = -3x^2 +$

$-3x(x-2) = 0$

$x = 0, 2$

$f'(x)$: − + −

x: 0 2

critical #: x = 0, 2

incr $(0, 2)$

decr $(-\infty, 0)$

$(2, \infty)$

rel max $(2, 4$

rel min $(0, 0$

5) $f(x) = (x - 1)^{\frac{2}{3}}$

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6) $f(x) = \frac{x^2}{x^2 - 9}$

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Find the points of inflection and discuss the concavity of the graph of the function.

7) $f(x) = 2x^3 - 6x^2 - 12x$

$f'(x) = 6x^2 - 12x - 12$

$f''(x) = 12x - 12$

$f''(x)$: − +

x: 1

concave down $(-\infty, 1)$

concave up $(1, \infty)$

POI $(1, -16)$

8) $f(x) = x(x - 4)^3$

$f'(x) = x \cdot 3(x-4)^2(1) + (x-4)^3(1)$

$= 4(x-4)^2(x-1)$

$f''(x) = 4(x-4)^2(1) + (x-1)(4)(2)(x-4)$

$= 12(x-4)(x-2)$

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9) $f(x) = \frac{x}{x^2+1}$

$f'(x) = \frac{(x^2+1)(1) - x(2x)}{(x^2+1)^2} = \frac{-x^2+1}{(x^2+1)^2}$

$f''(x) = \frac{(x^2+1)^2(-2x) - (-x^2+1)(2)(x^2+1)(2x)}{(x^2+1)^4} = \frac{2x(x^2+1)(x^2-3)}{(x^2+1)^4}$

10) $f(x) = \sin\frac{x}{2}, \quad [0, 4\pi]$

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see n

5. $f(x) = (x-1)^{2/3}$

$f'(x) = \frac{2}{3}(x-1)^{-1/3} = \frac{2}{3\sqrt[3]{x-1}}$

critical value $x = 1$

$f'(x)$: $-$ | $+$

x: 1

incr $(1, \infty)$

decr $(-\infty, 1)$

rel min $(1, 0)$

6. $f(x) = \frac{x^2}{x^2-9}$

$f'(x) = \frac{(x^2-9)(2x) - (x^2)(2x)}{(x^2-9)^2} = \frac{-18x}{(x^2-9)^2}$

critical values $x = 0, 3, -3$

$f'(x)$: $+$ | $+$ | $-$ | $-$

x: -3 0 3

incr $(-\infty, -3)$

$(-3, 0)$

decr $(0, 3)$

$(3, \infty)$

rel max $(0, 0)$

8. continued ...

$+$ | $-$ | $+$

2 4

concave up $(-\infty, 2)$ $(4, \infty)$

concave down $(2, 4)$

POI $(4, 0)$ and $(2, -16)$

9. continued ...

$-$ | $+$ | $-$ | $+$

$-\sqrt{3}$ 0 $\sqrt{3}$

concave up $(-\sqrt{3}, 0)$ $(\sqrt{3}, \infty)$

concave down $(-\infty, -\sqrt{3})$ $(0, \sqrt{3})$

POI $(-\sqrt{3}, -\sqrt{3}/4)$

$(0, 0)$

$(\sqrt{3}, \sqrt{3}/4)$

10. $f(x) = \sin\frac{x}{2}$ $[0, 4\pi]$

$f'(x) = \frac{1}{2}\cos\left(\frac{x}{2}\right)$

$f''(x) = -\frac{1}{4}\sin\left(\frac{x}{2}\right)$

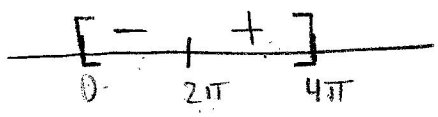
$-\frac{1}{4}\sin\left(\frac{x}{2}\right) = 0$

$\sin\left(\frac{x}{2}\right) = 0$

$\frac{x}{2} = \sin^{-1}(0)$

$\frac{x}{2} = 0, \pi, 2\pi$

$x = 0, 2\pi, 4\pi$



concave down $(0, 2\pi)$

concave up $(2\pi, 4\pi)$

POI $(2\pi, 0)$