

Differential Equations - Separation of Variables

1. a) $\frac{dy}{dx} = \frac{x}{y}$

$$\int y dy = \int x dx$$

$$\frac{1}{2} y^2 = \frac{1}{2} x^2 + C$$

$$y^2 = x^2 + C$$

$$\boxed{y = \pm \sqrt{x^2 + C}}$$

b) $\frac{dr}{ds} = 0.05 r$

$$\int \frac{dr}{r} = \int 0.05 ds$$

$$\ln|r| = 0.05s + C$$

$$e^{\ln|r|} = e^{0.05s + C}$$

$$\boxed{r = C e^{0.05s}}$$

c) $(2+x)y' = 3y$
 $(2+x) \frac{dy}{dx} = 3y$

$$\int \frac{dy}{y} = \int \frac{3}{2+x} dx$$

$$\ln|y| = 3 \ln|2+x| + C$$

$$e^{\ln|y|} = e^{3 \ln|2+x| + C}$$

$$\boxed{y = C (2+x)^3}$$

d) $y \cdot y' = \sin x$

$$\int y dy = \int \sin x dx$$

$$\frac{1}{2} y^2 = -\cos x + C$$

$$y^2 = -2\cos x + C$$

$$\boxed{y = \pm \sqrt{-2\cos x + C}}$$

e) $\frac{\sqrt{1-4x^2} y'}{\sqrt{1-4x^2}} = x$
 $\frac{dy}{dx} = x$

$$\int dy = \int \frac{x}{\sqrt{1-4x^2}} dx$$

$$u = 1-4x^2$$

$$\frac{du}{dx} = -8x$$

$$-\frac{1}{8} du = x dx$$

$$-\frac{1}{8} \int \frac{du}{\sqrt{u}}$$

$$-\frac{1}{8} \cdot 2u^{1/2} + C$$

$$\boxed{y = -\frac{1}{4} \sqrt{1-4x^2} + C}$$

f) $y \ln x - x y' = 0$
 $y \ln x - x \frac{dy}{dx} = 0$

$$-x dy = -y \ln x dx$$

$$x dy = y \ln x dx$$

$$\int \frac{dy}{y} = \int \frac{\ln x}{x} dx$$

$$\ln|y| = \int \frac{\ln x}{x} dx$$

$$u = \ln x$$

$$\frac{du}{dx} = \frac{1}{x}$$

$$du = \frac{1}{x} dx$$

$$\int u du$$

$$\frac{1}{2} u^2 + C$$

$$e^{\ln|y|} = e^{\frac{1}{2} (\ln x)^2 + C}$$

$$\boxed{y = C e^{\frac{1}{2} (\ln x)^2}}$$

$$2. a. y \frac{dy}{dx} - e^x = 0$$

$$\int y dy = \int e^x dx$$

$$\frac{1}{2} y^2 = e^x + C$$

$$(0, 4): \frac{1}{2} (4)^2 = e^0 + C$$

$$8 = 1 + C$$

$$7 = C$$

$$\frac{1}{2} y^2 = e^x + 7$$

$$y^2 = 2e^x + 14$$

$$y = \sqrt{2e^x + 14}$$

$$b. y(x+1) + \frac{dy}{dx} = 0$$

$$\frac{dy}{y} = -y(x+1) dx \rightarrow \int \frac{dy}{y} = \int -(x+1) dx$$

$$\ln|y| = -\frac{1}{2}x^2 - x + C$$

$$(-2, 1) \quad \ln|1| = -\frac{1}{2}(-2)^2 - (-2) + C$$

$$0 = -2 + 2 + C$$

$$0 = C$$

$$\ln|y| = -\frac{1}{2}x^2 - x$$

$$y = e^{-\frac{1}{2}x^2 - x}$$

$$c. y(1+x^2) \frac{dy}{dx} - x(1+y^2) = 0$$

$$y(1+x^2) dy = x(1+y^2) dx$$

$$\int \frac{y dy}{1+y^2} = \int \frac{x dx}{1+x^2}$$

$$u = 1+y^2$$

$$\frac{du}{dy} = 2y$$

$$\frac{1}{2} du = y dy$$

$$\frac{1}{2} \int \frac{du}{u}$$

$$\frac{1}{2} \ln|u| + C$$

go to the top right for the rest of the work

* 2c) (cont)

$$\frac{1}{2} \ln(1+y^2) = \frac{1}{2} \ln(1+x^2) + \frac{1}{2} \ln 4$$

$$\ln(1+y^2) = \ln(1+x^2) + \ln 4$$

$$\ln(1+y^2) = \ln(4(1+x^2))$$

$$1+y^2 = 4(1+x^2)$$

$$y^2 = 4(1+x^2) - 1$$

$$y = \sqrt{3+4x^2}$$

$$3. \frac{dy}{dx} = \frac{-9x}{16y}$$

$$\int y dy = \int -\frac{9}{16} x dx$$

$$\frac{1}{2} y^2 = -\frac{9}{32} x^2 + C$$

$$\frac{1}{2} (1)^2 = -\frac{9}{32} (1)^2 + C$$

$$C = \frac{25}{32}$$

$$\frac{1}{2} y^2 = -\frac{9}{32} x^2 + \frac{25}{32}$$

$$16y^2 = -9x^2 + 25$$

or

$$9x^2 + 16y^2 = 25$$

or

$$y = \sqrt{\frac{25-9x^2}{16}}$$

$$\frac{1}{2} \ln(1+y^2) = \frac{1}{2} \ln(1+x^2) + C$$

$$(0, \sqrt{3}) \quad \frac{1}{2} \ln(1+\sqrt{3}^2) = \frac{1}{2} \ln(1+0^2) + C$$

$$\frac{1}{2} \ln(4) = \frac{1}{2} \ln(1) + C$$

$$C = \frac{1}{2} \ln(4)$$