

# Differential Equations — Growth & Decay

1. a)  $\frac{dy}{dx} = x+2$

$$\int dy = \int (x+2) dx$$

$$y = \frac{1}{2}x^2 + 2x + C$$

2. a)  $\frac{dQ}{dt} = \frac{k}{t^2}$

$$\int dQ = \int \frac{k}{t^2} dt$$

$$Q = -\frac{k}{t} + C$$

b.  $\frac{dy}{dx} = y+2$

$$\int \frac{dy}{y+2} = \int dx$$

$$\ln|y+2| = x + C$$

$$y+2 = Ce^x$$

$$y = Ce^x - 2$$

b)  $\frac{dN}{ds} = k(250-s)$

$$\int dN = \int k(250-s) ds$$

$$N = K(250s - \frac{1}{2}s^2 + C)$$

or

$$(N = k(250s - \frac{1}{2}s^2) + C)$$

c.  $y' = \frac{5x}{y}$

$$\int y dy = \int 5x dx$$

$$\frac{1}{2}y^2 = \frac{5}{2}x^2 + C$$

$$y^2 = 5x^2 + C$$

$$y = \pm \sqrt{5x^2 + C}$$

3. a)  $\frac{dy}{dt} = \frac{1}{2}t$

$$\int dy = \int \frac{1}{2}t dt$$

$$y = \frac{1}{4}t^2 + C$$

$$10 = \frac{1}{4}(0)^2 + C$$

$$10 = C$$

$$y = \frac{1}{4}t^2 + 10$$

d.  $y' = y\sqrt{x}$

$$\int \frac{dy}{y} = \int \sqrt{x} dx$$

$$\ln|y| = \frac{2}{3}x^{\frac{3}{2}} + C$$

$$e^{\ln|y|} = e^{\frac{2}{3}x^{\frac{3}{2}}}$$

$$y = Ce^{\frac{2}{3}x^{\frac{3}{2}}}$$

b)  $\frac{dy}{dt} = -\frac{1}{2}y$

$$\int \frac{dy}{y} = \int -\frac{1}{2} dt$$

$$\ln|y| = -\frac{1}{2}t + C$$

$$\ln|10| = -\frac{1}{2}(0) + C$$

$$C = \ln 10 \quad \text{on back}$$

e.  $(1+x^2)y' - 2xy = 0$

$$(1+x^2)dy = 2xy dx$$

$$\int \frac{dy}{y} = \int \frac{2x dx}{1+x^2}$$

$$u = 1+x^2$$

$$\frac{du}{dx} = 2x$$

$$du = 2xdx$$

$$\ln|y| = \ln|1+x^2| + C$$

$$y = C(1+x^2)$$

$$3b \text{ (cont)} \quad \ln|y| = -\frac{1}{2}t + \ln 10$$

$$e^{\ln|y|} = e^{-\frac{1}{2}t} e^{\ln 10}$$

$$\boxed{y = 10e^{-\frac{1}{2}t}}$$

$$4a) \frac{dy}{dx} = ky$$

$$\int \frac{dy}{y} = \int k dx$$

$$y = Ce^{kx}$$

$$y_4 = Ce^{k(0)}$$

$$y = 4e^{kx}$$

$$10 = 4e^{k(3)}$$

$$\frac{5}{2} = e^{3k}$$

$$\ln \frac{5}{2} = \ln e^{3k}$$

$$\ln \frac{5}{2} = 3k$$

$$k = \frac{\ln \frac{5}{2}}{3}$$

$$y = 4e^{\frac{\ln \frac{5}{2}}{3}(6)} = \boxed{25}$$

$$b) \frac{dV}{dt} = KV$$

$$\int \frac{dV}{V} = \int K dt$$

$$V = Ce^{kt}$$

$$20000 = Ce^{k(0)}$$

$$20000 = C$$

$$V = 20000e^{kt}$$

$$12500 = 20000e^{4k}$$

$$.625 = e^{4k}$$

$$\ln .625 = \ln e^{4k}$$

$$\ln .625 = 4k$$

$$5. \frac{dT_{obj}}{dt} = k(T_{obj} - T_{med})$$

$$\frac{dT}{dt} = k(T - 80)$$

$$\int \frac{dT}{T-80} = \int k dt$$

$$e^{\ln |T-80|} = e^{kt+C}$$

$$T-80 = Ce^{kt}$$

$$T = 80 + Ce^{kt}$$

$$(0, 1500): 1500 = 80 + (Ce^{k(0)})$$

$$1420 = C$$

$$T = 80 + 1420e^{kt}$$

$$(1, 1120): 1120 = 80 + 1420e^{k(1)}$$

$$\ln \frac{52}{71} = \ln e^k$$

$$y = 20000e^{\frac{\ln \frac{52}{71}}{4}(6)} \approx \boxed{9882.118}$$

$$T = 80 + 1420e^{\ln \frac{52}{71}(5)}$$

$$\approx \boxed{379.236^\circ}$$