Practice 6.2 Volume: The Disk Method

1) Set up and evaluate the integral that gives the volume of the solid formed by revolving the region about the x-axis.

a) $y = -x + 1, x = 0, y = 0$

$$\pi \int_0^1 (-x+1)^2 dx = \frac{\pi}{3}$$

b) $y = \sqrt{x}, \quad x = 1, x = 4, y = 0$

$$\pi \int_1^4 (\sqrt{x})^2 dx = \frac{15\pi}{2}$$

2) Set up and evaluate the integral that gives the volume of the solid formed by revolving the region graphed in the first quadrant about the y-axis.

a) $y = x^2, y = 0, y = 4, x = 0$

$$\pi \int_0^4 (\sqrt{y})^2 dy = 8\pi$$

b) $y = x^{2/3}, x = 0, y = 1$

$$\pi \int_0^1 (y^{3/2})^2 dy = \frac{\pi}{4}$$

3) Find the volume of the solid generated by revolving the region bounded by the graphs of the equations $y = \sqrt{x}, \ y = 0$, and $x = 4$ about the following lines.

a) the x-axis

$$\pi \int_0^4 (\sqrt{x})^2 dx = 8\pi$$

b) the line $x = 4$.

$$\pi \int_0^2 (4-y^2)^2 dy = \frac{256\pi}{15}$$

4) Find the volume of the solid generated by revolving the region bounded by the graphs of the equations $y = x \ \ y = 0, \ y = 4$ and $x = 6$ about the line $x = 6$.

$$\pi \int_0^4 (6-y)^2 dy = \frac{208\pi}{3}$$

5) Find the volume of the solid generated by revolving the region bounded by the graphs of the equations $y = \frac{1}{\sqrt{x+1}}, \ y = 0, \ x = 0$ and $x = 3$ about the x-axis.

$$\pi \int_0^3 \left(\frac{1}{\sqrt{x+1}}\right)^2 dx \approx 4.355$$

6) Find the volume of the solid generated by revolving the region bounded by the graphs of the equations $y = \sin x, \ y = 0, \ x = 0$ and $x = \pi$ about the x-axis.

$$\pi \int_0^\pi (\sin x)^2 dx = \frac{\pi^2}{2} \approx 4.935$$