Precalculus Unit 4 Homework—Modeling with Exponential and Logistic Functions

In exercises 1-3, use the data in the table and assume that the growth is exponential.

city	1990 population	2000 population
Austin, Texas	465,622	656,562
Columbus, Ohio	632, 910	711,470

1) When will the population of Columbus surpass 800,000 persons? 2010
2) When will the populations of the cities be equal? 741,862 in 2004
3) Which city will reach a population of 1 million first? In what year does that occur? 2012

4) Using 20th-century U.S. census data, the population of Ohio can be modeled by $P(t) = \dfrac{12.79}{1+2.402e^{-0.0309x}}$ where P is the population in millions and t is the number of years since 1900. Based on this model, when was the population of Ohio 10 million?

   69.67 (1969)

5) Using 20th-century U.S. census data, the population of New York state can be modeled by $P(t) = \dfrac{19.875}{1+57.993e^{-0.035005t}}$ where P is the population in millions and t is the number of years since 1800. Based on this model,
   a) What was the population of New York in 1850? 1.79 million
   b) What will New York state's population be in 2010? 19.16 million
   c) What is New York's maximum sustainable population (limit to growth)? 19.875 milli

6) The number B of bacteria in a petri dish culture after t hours is given by $B = 100e^{0.693t}$.
   a) What was the initial number of bacteria present? 100
   b) How many bacteria are present after 6 hours? 6394

7) The amount C in grams of carbon-14 present in a certain substance after t years is given by $C = 20e^{-0.0001216t}$.
   a) What was the initial amount of carbon-14 present? 20 grams
   b) How much is left after 10,400 years? 5.647 grams
   c) When will the amount left be 10 g? 5700.22 years

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8) Find the bank account balance if the account starts with $11, has an annual rate of 4%, and the money is left in the account for 12 years. $160.10

9) Bacteria can multiply at an alarming rate when each bacteria splits into two new cells, thus doubling. If we start with only one bacteria which can double every hour, how many bacteria will we have by the end of one day?

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10) You drink a beverage with 120 mg of caffeine. Each hour, the caffeine in your system decreases by about 12%. How long until you have 10 mg of caffeine in your system?

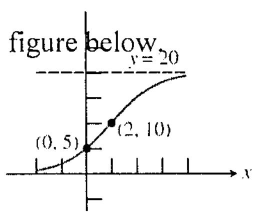
19.44 hr

11) Find a logistic function of the form: $f(x) = \dfrac{c}{1 + a \bullet b^x}$ satisfying the following conditions:

Initial value = 10, limit to growth = 40, passing through (1, 20)

$$f(x) = \frac{40}{1+3(\frac{1}{3})^x}$$

12) Determine a formula for the logistic function of the form: $f(x) = \dfrac{c}{1 + a \bullet b^x}$ whose graph is shown in the figure below.



$$f(x) = \frac{20}{1+3(\frac{1}{\sqrt{3}})^x}$$

13) The number of students infected with the swine flu at HSHS after t days is modeled by the function $f(t) = \dfrac{800}{1 + 49 \bullet e^{-0.2t}}$.

a) How many students were sick when the outbreak started? 16

b) When will the number of infected students be 200? 13.97 days

c) What is the maximum number of students who could be infected? 800

14) Suppose that an experimental population of fruit flies increases exponentially. The population began with 100, and after 2 days the population reached 300 flies.

a) Write a model, **$P(t)$**, to represent the situation. $p(t) = 100(\sqrt{3})^t$

b) How many flies will be present in 10 days? 24300

c) How long will it take for the population to reach a billion? 29.34 days