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1970 BC 7

Solution

$$(a) \quad f(x) = \frac{x - \sin 2x}{\sin x} = \frac{x}{\sin x} - 2 \cos x, \text{ so } \lim_{x \rightarrow 0} f(x) = 1 - 2 = -1$$

$$\text{Or } f(x) = \frac{x - \left(2x - \frac{8x^3}{6} + \dots\right)}{x - \frac{x^3}{6} + \dots} = \frac{-1 + \frac{4}{3}x^2 - \frac{4}{15}x^4 + \dots}{1 - \frac{x^2}{6} + \frac{x^4}{120} - \dots} \rightarrow 1 \text{ as } x \rightarrow 0.$$

$$\text{Or } \lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} \frac{1 - 2 \cos 2x}{\cos x} = -1$$

Therefore defining $f(0) = -1$ will make f continuous for all x in the interval $-1 < x < 1$.

$$(b) \quad f'(0) = \lim_{h \rightarrow 0} \frac{f(0+h) - f(0)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\frac{h - \sin 2h}{\sin h} + 1}{h}$$

$$= \lim_{h \rightarrow 0} \frac{h - \sin 2h + \sin h}{h \sin h} \quad \left[= \frac{0}{0} \right]$$

$$= \lim_{h \rightarrow 0} \frac{1 - 2 \cos 2h + \cos h}{h \cos h + \sin h} \quad \left[= \frac{0}{0} \right]$$

$$= \lim_{h \rightarrow 0} \frac{4 \sin 2h - \sin h}{-h \sin h + 2 \cos h} = \frac{0}{2} = 0$$

Therefore $f'(0)$ exists and has the value 0.

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1991 AB4
Solution

(a) $f(x) = 0 \Leftrightarrow |x| = 2, x \neq 2$

$x = -2$

(b) For $x \geq 0, x \neq 2$

$$f(x) = \frac{x-2}{x-2} = 1$$

Therefore $f'(1) = 0$

(c) For $x < 0, f(x) = \frac{-x-2}{x-2}$

$$f'(x) = \frac{(x-2)(-1) - (-x-2)(1)}{(x-2)^2} = \frac{4}{(x-2)^2}$$

Therefore $f'(-1) = \frac{4}{9}$

(d) $-1 < y \leq 1$